Ergodicity, KAM, FPUT

Boltzmann introduced the microcanonical ensemble in 1868, [1], and immediately attempted to give an example of a system whose stationary states would be described by the emsemble (as suggested also by his ergodic hypothesis). The example, [2], has been recently shown to be incorrect, if taken literally: the point was to suppose that constants of motion, if any besides the energy, would necessarily be smooth functions; and soon later he warned on the dangers implicit in a similar assumption. Fifty years later Fermi wrote a paper attempting to prove that in general a nonlinear system should be ergodic, [3]: but his proof relied again on Boltzmann's assumption. Thirty-four more years elapsed, and Fermi returned on the problem collaborating with Pasta, Ulam, Tsingou: the surprise was that the considered non linear chain was apparently not following the ergodic hypothesis. In the same year Kolmogorov had proved the conservation of many quasi periodic motions in nonlinear perturbations of integrable systems, [4]: his theorem was considered, already a few years later, a possible explanation of the FPUT work, [5]. This was only the beginning of intense research: here a brief sketch is presented to illustrate the above themes and the connection with the multiscale aspects of the problems, and the "Renormalization group method" intended as a map $\mathcal{R}$ whose iterations can be interpreted as successive magnifications, zooming on ever smaller regions of phase space in which motions develop closer and closer to the searched quasi periodic motion of given spectrum.