Preface to the second edition

The book has been entirely revised and includes several corrections. I kept the informal style, as a choice to avoid that first readers about chaos give up considering the proposed theories as too mathematical.

The first Chapter gives an historical perspective and a motivation for the classic works partially translated in Ch.6. The "heat theorem" section 1.4 is new and presents in modern form the proof of Boltzmann in Clausius' version: it replaces Boltzmann's elementary example, now shifted to appendix (A). In the historical part the interpretation of Boltzmann's example on the heat theorem in Sec. (6.3) is updated and includes the new results.

Chapters 2,3,4 are an attempt to a consistent presentation of the main ideas and tools that were developed since the 1970's toward the intrepretation of the new experimental works and simulations involving chaotic motions. The ideas are founded on the ground breaking work by Sinai, Bowen and Ruelle and on the need to interpret the results of the simulations performed on the new computer hardware since it became available to "every one". I have tried to stress, while illustrating several examples, that stationary non equilibrium and equilibrium can be viewed from a unifying viewpoint. Many changes have been apported: in the years since the first edition I realized the need of a substantial revision: still I refrained from adopting a really formal style. Formal expositions exist and are always quoted: but my intention was to introduce the problems while communicating the surprise that I felt at the unification of basic theoretical ideas of Statistical Mechanics, Dynamical Systems, Fluids, Probability Theory and Computer simulations.

The main changes affect part of Chapter 4, Chapter 5 (on conjectures and suggestions), and the Appendices. I kept the heuristic and phenomenological character, with several variations due to the evolution of the subjects since the first edition: several sections have been rewritten and the order of presentation has been changed. The long appendices on the BBGKY hierarchy, whose purpose was to formulate, as an open problem, some difficulties on its use to describe a nonequilibrium stationary state, have been eliminated because the approach resulted, in the meantime, too ambitious (*i.e.* the problem did not progress). Likewise the appendices on the pendulum subject to noise, have also been suppressed: the proposal was to present a possible path to the solution of a problem on the convergence of a formal series representing the stationary state. Since then, new results have been obtained following a different path. The theory of Lyapunov exponents pairing, appendix I, has been written following the original derivations and only mentioning the simpler, successive, ones. An appendix on the large deviations theorem for SRB distributions is added for completeness, N.

The part dedicated to the fluids (viscous, incompressible and with periodic boundary) in Sec. 4 is deeply changed and takes into account recent ideas, on

the regularized Navier-Stokes equations: with entirely new conjectures (and related simulations) on the properties that might survive the regularization removal.

The applications to fluid mechanics, in Chapter 5 (Sec.,4,5,6) and appendices J,K,L also replace and update the old versions. Comparison between the Navier-Stokes equation and reversible equations associated with it and with the Euler equation, is illustrated via several conjectures, or heuristic arguments: with special attention to the possible equivalence between irreversible and reversible equations.

The continuing debate about reversibility and irreversible behaviour is repeatedly commented, making also use of heuristic digressions.

Irreversibility in systems governed by reversible equations and reversibility in systems governed by irreversible equations are interesting when the equations model the same physical system: the resulting conflict is discussed when possible, not only in fluids but also in mechanical or more general dynamical systems.

The above are changes which are remarkable at first sight: but innumerable changes have been implemented where I felt that the previous version was incomplete or misleading or, simply, I thought it could be improved.

There are several sources of errors hidden, in the literature, in fast applications of mathematical statements, for instance in applications of the fluctuation theorem: I have tried to put them in evidence, and to make them more clear by frequent reference to other book sections where the same problems arise.

The main message that I would like to deliver is that this work is intended to stress, via an informal or heuristic style, the breakthrough proposal by Ruelle to unify the study of stationary nonequilibrium and of equilibrium via the paradigm offered by the simplest chaotic systems (namely the Anosov systems): seen as playing for disordered motion the role that the simplest ordered systems play for ordered motions (namely the systems of harmonic oscillators, *i.e.* systems animated by quasi periodic motions, or more generally by integrable systems).

Giovanni Gallavotti
Roma, 25 November 2024