Lucio Russo: On Probability Theory, and Current Interests

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*Keywords:*Percolation, Probability, Ising Model, Phase Coexistence, Coding theory, Markov Processes, Hellenistic Science

Abstract: A review of Russo's original contributions to various fields of Probability theory and of his parallel interest on History and influence of Hellenistic Science

The work of Lucio Russo started in the early 1970's: I met him in Napoli and it became immediately clear that I had met a very young and very promising scientist. And I had the privilege to collaborate with him on the problem of the isomorphism between Ising model equilibrium probability distributions in the convergence domain of the cluster expansion and Bernoulli schemes of the same entropy, di Liberto et al. (1973).

Soon afterwards I left Napoli: but I continued to follow his work. His study of the isomorphism problem was pursued and led to a major result on coding theory; he proved the existence of a finitary code between the simplest Markov process (*ie.* the one dimensional Ising model with nearest neighbor interaction) and the Bernoulli shift with the same entropy, Monroy and Russo (1975). I saw the impact and the recognition that this work had on the complete solution of the general construction (by Keane and Smorodinsky, Keane and Smorodinsky (1977)) of a code of a Markov chain into a Bernoulli shift.

To perform these works he learned in a short time the deep and innovative work of Ornstein on the theory of Bernoulli shifts and was able to give a substantial contribution to the applications of the theory.

He rapidly became internationally known as a leading probabilist: contributing to advances on the percolation problem beginning with studying the attractive Ising model, establishing a close relation between the existence of infinite clusters of spins + or − and the presence of spontaneous magnetization in the extremal states $\mu_\pm$, with the remarkable result that in general dimension the spontaneous magnetization in the states μ_+ or μ_- yields a lower bound to the percolation probability (*ie.* existence of infinite clusters of + or − respectively). Furthermore in $2D$ coexistence of infinite clusters of both types was excluded in all pure states, Coniglio et al. (1976, 1977).

In the course of about three years he worked intensely on percolation (in Bernoulli sites distribution as well as in Ising model equilibria) employing the FKG and GHS inequalities: first extending a result by Harris on critical percolation and proving that mean size of the finite clusters is finite, Russo (1978). This led, in a subsequent work, to the proof, in the case of percolation on Z^2 of the Essam-Sykes conjecture on the critical probabilities of two "matching graphs" ($p_c + p_c^* = 1$), Russo (1981).

A result on the difficult problem of the (site) percolation in Z^3 followed after a concatenated series of studies (mostly in $2D$). First on a proof that in $2D$-Ising there is a unique infinite cluster for $T \leq T_c$ and coincidence of the critical point and the percolation point (at difference from their already known non-coincidence in $3D$), Coniglio et al. (1976, 1977); the smoothness, away from the critical point, of the $2D$ percolation probability dependence on the occupation probability followed, Russo (1978); the Essam-Sykes conjecture was then proved, Russo (1981), obtaining also (on Z^2) that the site percolation probability is a continuous function of the site probability p. Finally the methods, always based on inequalities known in statistical mechanics and on several extensions developed in Russo's works, have been applied to the $3D$-percolation and to the $3D$-Ising model: the critical site percolation is proved to be $< \frac{1}{2}$ and in $3D$-Ising at high temperature (and small field) coexisting infinite clusters are shown to be possible, Campanino and Russo (1985).

The first results on percolation left way for the groundbreaking result, based on the Ising model inequalities and relying also on ideas developed in the first few percolation papers (which will also be further developed in the later works), on the uniqueness of the translation invariant Gibbs states in the 2 dimensional Ising model, Russo (1979): it introduced a fresh view and a new method on the uniqueness problem of the pure phases and proved that the states that were translation invariant in one direction were necessarily invariant under all translations. Immediately after publication of his work his result has been basic to the final solution of the long standing conjecture about the translation invariance of the equilibrium states of the Ising model at all temperature and field values (independently in Higuchi (1981); Aizenman (1980)).

Percolation continued to be the focus of his interests; for the site percolation problem, with site probability p, Russo proved that the probability of a given positive event A measurable at infinity (or close enough to be such) passes from 0 to 1 as p grows through a value p_0: an extension of the

Hewitt-Savage zero-one law, Russo (1982). A result also shown to be useful to determine, by a new method, the relation $p_c + p_c^* = 1$ for the critcal percolations on the square lattice and, respectively, on the matching lattice.

A theory of a different kind of percolation, the (independently occupied with probability p) plaquettes percolation on $Z^d, d \geq 3$ is studied. It was shown that in $d = 3$ the existence of a phase transition in the dependence of the probability that the occupied plaquettes form a surface with given boundary decays exponentially with the area of the surface for small p but at high plaquettes density, *ie.* p close to 1, the probability decays with the length of the boundary. The result, Aizenman et al. (1983), is obtained employing also the duality between the plaquette percolation and the bond percolation and the sharpness of the transition (as a function of p) is related to a conjecture on the $3D$ bond percolation.

The problem of the uniqueness of infinite percolating clusters is then reexamined first in the case of Bernoulli percolation and then in the $2D$ case of rather general site distribution, subject to conditions of translation ergodicity in each direction and to the monotonicity of $\mu(F \cap G) \geq \mu(F)\mu(G)$, Gandolfi et al. (1988a,b).

The attention to inequalities, an essential feature of most if not all Russo's works, has generated a work on new inequalities related to the FKG inequalities which have new applications to a variant of the Ising model, the "plaquette model", Cammarota and Russo (199), considered in certain gauge theories.

Russo has also contributed to other fields of Mathematics like dynamical systems and analysis, with the same uncompromizing attention to mathematical rigor transpiring from his probability works, providing insights and suggestions in the normal seminars held in the departments where he has been a member, Franceschini and Russo (1981); Facchinetti and Russo (1983).

All the above results were obtained by Russo working alone or in collaboration with other scientists. His sharp understanding of the probability theory of Bernoulli distributions or of Ising distributions as been always an essential contribution. At the end of the 1980's Russo's main interests switched to the history of Science revealing a less known aspect of his personality which had remained hidden to most of his colleagues. In a sense he really concentrated on the classic achievements from the Hellenistic science to the 1600's.

This choice left little room to continue developing probability theory of percolation (whether Bernoulli or Ising or other): I witnessed that this gen-

erated deep regrets from several leading probabilists.

His involvement in the new task that he set to himself was totalizing: to study the few remnants of the Hellenistic age he ended up gradually learning classic Greek (starting from brushing up what in Latin and Greek he had learned in high school). The works that arise from this activity are for me difficult to comment, not having really worked on ancient and modern history of Science, and I try mention briefly some of them: many are in Italian and I expect that after translation into English they will generate an even wider debate.

He began, $\sim$1990, with a critique of the authenticity of the geometric entities in the definitions at the beginning of Euclid's elements: the analysis found its way in a paper on the "Bollettino dei classici" of the Accademia dei Lincei, before a new English version appeared on the "Archive History Exact Sciences", Russo (1992, 1994). The thesis is that, in the several centuries elapsed since Euclid to the present version of the Elements, the introductory definitions were added to the original work, or 'simplified'. The thesis is developed by a detailed logical analysis of the definitions: they are certainly captivating and lead to meditate on the subject. They are often not accepted on the grounds that some incompetent scholar should have dared to 'simplify' Euclid's work, see MR16127169. Personally the idea of a not literal transmission of the definitions is not *a priori* unacceptable: the example of Boltzmann's work (whose original sources are still available) is a compelling example of how scientific ideas may be misinterpreted, and nevertheless be useful, just a few years (not even centuries) after most scientists proceeded to make use of their consequences relying on expositions rather than relying on the original.

Several essays followed: with sharp critiques of modern interpretations of Hellenistic achievements in Astronomy, Geography, Mechanics, Dynamics (Russo (1993a, 1994)), gravitation and tidal theory (Russo (1993b,c)), or where the analysis of the accusation of "impiety" to Aristarchus because of his heliocentric system is (convincingly) confuted, Russo and Medaglia (1996).

And the essays have been often collected and extended in books, starting with the "Forgotten Revolution" (1996), Russo (1996), whose initial Italian version has been revised and translated into English (2003), Russo (2004), and later into German. The book is captivating, with several insights into the interpretations of Greek and Roman texts which are often very convincing (showing the importance that, sometimes, leading scientists become involved

in historical work). The thesis that the development of Science at the Renaissance was due to the study by the main scientists of the time, who might have not acknowledged sources possibly lost after consultation by then, of old manuscripts still available after the fall of Constantinople, has met a rather strong reaction as is witnessed in the reviews MR2038833, (or even rejected see MR1629047), by other historians: see also the reviews on Nature, 430 (5 August 2004), and on Physics World (April 2004),or Notices of AMS (May 1998). Even if the analysis dealing with Renaissance, Modern or Contemporary might be unsatisfactory to some, the discussion of the classic achievements and of the imperial decay of Science is captivating and rich of new insights that one can hardly stop reading.

It has been and remains a stimulating work whose theses will continue to be considered. An example that I find illuminating of the kind of critical examination of fonts (here Pliny and Vitruvius) is the Sec.10.6,p.319 (Italian, 2d ed.) or Sec.10.8, p.296, (English) on the Moon motion. Also appearing in other papers: for instance in an essay on Hipparchus' astronomy, Russo (1994), and other pre-Ptolemaic sources.

Russo has since written more essays, eventually coherently organized and collected into books. For instance the book "Flussi e riflussi" on the history and the origins of the theory of tides, expanding Russo (1995), gives a detailed and documented analysis of the developments in the Modern period and its roots in Antiquity, Bonelli and Russo (1996); Russo (2003), stressing that the theory was developed with apparently independent contributions repeatedly discovered and lost or forgotten, influencing many thoughts and ending to be eventually attributed only to particular scientists (Newton, Laplace, Kelvin, G. Darwin). And, to list only a few essays, a discussion on the nature and relation between Mathematics and Physics and its evolution from Hellenism to contemporary science, or a paper on the "case" of Aristarchus, Russo (2002), which goes back to the impiety accusation and clarifies its unfoundedness, shown to be due to a misunderstanding of a text of Plutarch; here at the same time are commented the important contributions of Seleucus to heliocentrism and to tides theories, Russo (2002); and an essay on Archimedes and on the myths about him, Russo (2013b).

A second (unorthodox) book "L'America dimenticata", Russo (2013a), develops ideas whose birth can be traced to the quoted "Forgotten Revolution" on the discovery of America by Carthaginians (at least) before and after their dispersion to flee the Roman victory. The book is centered on the thesis that civilization is not deterministically controlled by the human genes, and

independent human communities might evolve very differently rather than proceeding, deterministically, in parallel through the same stages, only up to random time delays. In particular major calamities might completely stop, pull back by centuries or divert the evolution: one of them is the Roman destruction of Carthage which cut earlier links between Europe and Central America, as Russo contends, and caused the loss of the knowledge of the Atlantic navigation practiced at least by Phoenicians and Carthaginians which established a link between the evolution of the European and American civilization of which remain scanty vestiges.

The case is discussed in detail, expanding Russo (2013c), and catching the opportunity to analyze a debated geography question and to offer a new interpretation of the related work of Ptolemy. The highlight is a scientific analysis of the reduction of the size, from 252000 to 180000 stadia, of the Earth equator: from the value established by Eratosthenes and used by Hipparchus to the value documented, more than three centuries later, by Ptolemy.

It is argued that the problem is not due, as often deemed, to the error of Ptolemy on the value of "Eratosthenes stadion" (erroneously taken, by Ptolemy, to be ~ 1.43 factor smaller than that used afterwards, way into the next millennium). The error, as Russo argues, can be traced to the identification of the extreme western site of the inhabited world with the Canary isles.

In Ptolemy the latitude of the extreme western site was $\sim 5.5^o$ and the longitude width $\sim 1^o$ while the longitude is that of the Canaries The latter, however, are 15^o more north in latitude, form a $\sim 5^o$ wide archipelago and are $\sim 45^o$ east of the Lesser Antilles. The latter instead fit in latitude, longitude width and longitude with the Hellenistic Fortunate Isles if the eastern and western limits of the inhabited world are supposed to be at 180^o degrees from each other and the eastern is fixed at the known distance from the longitude of the capital of China (Ptolemy actually uses an unidentified city that Russo reasonably places close to its historical location (Xian) around 150BC).

Russo's conclusion and proposal is that Ptolemy assumed *a priori* the width of the inhabitable world to be 180^o, a datum apparently legated by earlier scientists (and evidence is analyzed), with the same eastern limit but advanced eastwards the western limit, the reason being that the Fortunate Isles had been confused with the Canaries Isles, having been forgotten after the events following the Punic war (further arguments can be found in Russo (2016)). A consequence seems to be that Ptolemy had the correct distances along (some) parallels but no accurate astronomical determination of longi-

tudes for at least a few known sites [1] to fit the known distances: giving the Canaries as extreme limit and 180^o degrees width of the inhabited world a shorter Earth radius and a dilation of the longitudes resulted.

The analysis also fixes anoother puzzle left by Ptolemy's Geography, besides the length of the equator: the location of Tule as an island above Norway is, after Russo's analysis, relocated (renormalizing Ptolemy's value of the longitude) to a point on the east coast of Greenland consistent with the reported discoveries of Pitia's explorations.

The book contains a dense set of footnotes to document the statements taken from the literature (mostly classical) and a captivating analysis of them. Unfortunately the notes have been relegated to the end of the chapters (as done also in the next book by Russo: a choice increasingly practized by printers, but) very inconvenient for the readers because the footnotes are really essential here.

In a third book "Stelle, atomi e velieri" ("Stars, atoms and sailing ships"), Russo (2015), the basic idea on the non deterministic evolution of science and its dependence on continuous interaction, through possibly tenuous temporal or spatial links between different scientists, is taken up again and further developed. Here Russo puts together all his previous arguments and offers a rapid and synthetic view of some of the major events in the history of Science always paying attention and adding evidence to reinforce the main idea that can be found in his earlier books and papers. For instance it is interesting to see collected and organized in a wider context the analysis of the role of Copernicus and Bruno and the violent opinions of Voltaire on Aristarcos, which will make at least some readers wish that Voltaire had been more careful in his judgment, see also Bonelli and Russo (1996); Russo (2002). The book expands all themes treated earlier, adding substantial remarks on Science after the Renaissance, with arguments in favor of one of his main theses, *ie.* the cultural interdependence between many scientists, starting with Hellenic and Hellenistic science, and that major discoveries are often only attributed to "giants". He acutely examines the evolution of the disciplines called Mathematics, Physics and Mathematical Physics expressing judgments which will, likely, generate lively debates.

Recently Russo has proposed a probabilistic approach to the determina-

[1] Longitude could have been only determined by on site astronomical observations. Which require (good instruments and) a physical presence to observe the exact moment of an expected celestial event, *e.g.* a lunar eclipse or an equinox, and either the exact local solar hour or the concomitant equatorial coordinates of a star. Such observations were possible in antiquity (and positively mentioned and appreciated by Ptolemy, (Russo, 2013a, p.158)) but of which there are few detailed records, if any.

tion of the date of events known to have occurred in a time interval, Isola and Russo (2016): a return to probability theory, applied to the birth-date of Hypatia and Theon. A further book is dedicated to the history of Science in Italy, Russo and Santoni (2010). The editorial activity of Russo led him also to write, perhaps under the influence of his personal history with parents deeply dedicated to high school teaching (hence with an early direct knowledge of the problems and issues) an essay on education criticizing, often harshly, the modern methods of teaching science in the Italian high schools, Russo (1998): a critique which hopefully will be heard, soon or later.

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