timing event

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< Chaotic hypothesis
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Mathematical models for time evolution can be differential equations whose solutions represent motions developing in continuous time $t$ or, often, maps whose $n$-th iterate represents motions developing at discrete integer times $n$. The point representing the state of the system at time $t$ is denoted $s_t x$ in the continuous time models or, at the $n$-th observation, $S^n x$ in the discrete time models. Here $x, \xi$ will be points on a manifold $X$ or $\Xi$ respectively, called the phase space, or the space of the states, of the system.

The connection between the two representations of motions is illustrated by means of the following notion of "timing event." 

Physical observations are always performed at discrete times: i.e. when some special, prefixed, timing event occurs, typically when the state of the system is in a set $\Xi \subset X$ and triggers the action of a "measurement apparatus," e.g. shooting a picture after noting the position of a clock arm. If $\Xi$ comprises the collection of the timing events, i.e. of the states $\xi$ of the system which induce the act of measurement, motion of the system can also be represented as a map $\xi \rightarrow S \xi$ defined on $\Xi$.

For this reason mathematical models are often maps which associate with a timing event $\xi$, i.e. a point $\xi$ in the manifold $\Xi$ of the measurement inducing events, the next timing event $S \xi$.

If the system motions also admit a continuous time representation on a space of states $X \supset \Xi$ then there will be a simple relation between the evolution in continuous time $x \rightarrow s_t x$ and the discrete representation $\xi \rightarrow S^n x$ in discrete integer times $n$, between successive timing events, namely $S \xi \equiv S_{\tau(\xi)} \xi$, if $\tau(\xi)$ is the time elapsing between the timing event $\xi$ and the subsequent one $S \xi$.

The discrete time representation is particularly useful mathematically in cases in which the continuous evolution shows singularities: the latter can be avoided by choosing timing events which occur when the point representing the system is not singular nor too close to a singularity (when the physical measurements become difficult or impossible).

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