timing event

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< Chaotic hypothesis

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Mathematical models for time evolution can be differential equations whose solutions represent motions developing in continuous time t or, often, maps whose n-th iterate represents motions developing at discrete integer times n. The point representing the state of the system at time t is denoted $S_t x$ in the continuous time models or, at the n-th observation, $S^n\xi$ in the discrete time models. Here x,ξ will be points on a manifold X or Ξ respectively, called the phase space, or the space of the states, of the system.

The connection between the two representations of motions is illustrated by means of the following notion of ``timing event.

Physical observations are always performed at discrete times: i.e. when some special, prefixed, timing event occurs, typically when the state of the system is in a set $\Xi\subset X$ and triggers the action of a ``measurement apparatus, e.g. shooting a picture after noting the position of a clock arm. If Ξ comprises the collection of the timing events, i.e. of the states ξ of the system which induce the act of measurement, motion of the system can also be represented as a map $\xi\to S\xi$ defined on Ξ.

For this reason mathematical models are often maps which associate with a timing event ξ, i.e. a point ξ in the manifold Ξ of the measurement inducing events, the next timing event $S\xi$.

If the system motions also admit a continuous time representation on a space of states $X\supset\Xi$ then there will be a simple relation between the evolution in continuous time $x\to S_t x$ and the discrete representation $\xi\to S^n\xi$ in discrete integer times n, between successive timing events, namely $S\xi\equiv S_{\tau(\xi)}\xi$, if $\tau(\xi)$ is the time elapsing between the timing event ξ and the subsequent one $S\xi$

The discrete time representation is particularly useful mathematically in cases in which the continuous evolution shows singularities: the latter can be avoided by choosing timing events which occur when the point representing the system is not singular nor too close to a singularity (when the physical measurements become difficult or impossible).