Aspects of the ergodic, qualitative and statistical theory of motion

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Preface (2003)

This book started as a translation of an earlier Italian book by one of us. The present book is not only a substantial revision of the earlier book but it also deals with a number of problems that were not treated there. The main novelty is the systematic treatment of a few characteristic problems of ergodic theory by a unified method in terms of convergent power series expansions. The methods of resummation necessary to deal with such series have become familiar as "renormalization group methods" and in our opinion provide the simplest and most intuitive approach to the problems that we discuss (like KAM theory and Anosov maps).

A substantial number of questions, some even more interesting than the ones in the main body of the book, are treated in the form of guided problems. This is not done to save space (in spite of the importance of such saving) but mainly because we feel that it constitutes, for the interested reader, a more stimulating way of presenting the matter.

We are grateful to Dr. Alessandro Giuliani for his many critical comments and suggestions. And we warmly thank Professor Wolf Beiglböck for his continuous support and encouragement.

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Preface to the Italian book (1980)

In recent years books on problems of ergodic theory appeared with the common theme of stressing the close relations observed between it and the theory of Gibbs states.

In these lectures I collect the contents of several courses and seminars that I presented in various occasions (mainly Courant Institute, (1972), EPF of Lausanne (1979), Scuola di perfezionamento in Fisica di Roma (1980)).

The organization and the choice of the arguments has been largely influenced by the monographs of R. Bowen and D. Ruelle: this book mainly deals with the ergodic theory of maps (one dimensional ergodic theory). My intention has been to exhibit the existence of a tread connecting apparently different problems by always selecting, for illustration purposes, simple examples and by often avoiding formulating results in the widest generality: this collection of lectures is intended for beginners in ergodic theory and it follows a particular point of view: that of keeping always contact with statistical mechanics (from which the problems originated but which no longer appears clearly as the source of the main problems, ideas and conjectures).

The problems at the end of every section are an essential complement to the text and they are not always obvious or of easy solution. In using this book as a textbook for a course it would be useful to solve many of the problems leaving to the students the task of studying the main part of the textbook: it is only through the problems that the student can reach and go beyond the formal (hence superficial) level of understanding and to master the technical difficulties and to reach at the roots of the ideas. Almost all problems propose a proof of a classical result that, although I consider it not very useful to describe explicitly in the text, nevertheless cannot be neglected of skipped if one is attempting to study the subject of ergodic theory.

At the same time the problems provide a guide to the systematic study of related results or of results traditionally pertaining to other disciplines; the guide is meant to put within the frame of ergodic theory a certain number of them and to show their connection with some basic achievements in ergodic theory. Such a guide is necessary because the extraordinary variety of techniques and methods that constitutes one of the reasons of the fascination that ergodic theory exerts on mathematicians becomes also the main difficulty for a beginner.

The lectures on Gibbs states concern *only* one–dimensional systems: nevertheless a good part of the results extends with obvious modifications to systems in higher dimensions (which are more directly related to the statistical mechanics) and this holds in particular for the §(6.1)§(6.2) and to a fair extent for §(5.1), §(5.2), §(6.1), §(6.4), §(6.4). Therefore I think that what is discussed in this book could also serve as an introduction to the theory of Gibbs states and of stochastic processes in more dimensions.

In the references the publication years refer to the quoted edition rather than to the original edition.

While writing up these lectures I benefitted from several discussions with students and collaborators or colleagues whom I wish to thank because without their help and encouragement this work would have been impossible. In particular I thank G. Benfatto, M. Campanino, F. Ledrappier, G. Pianigiani.

I am also indebted to several institutions for invitations to give courses and seminars providing me the chance and the means to learn and to organize the topics treated here: among them in particular Institut Hautes Etudes Scientifiques (IHES), Scuola Matematica Internazionale (SMI), Istituto Nazionale di Alta Matematica (INDAM), Scuola di Perfezionamento in Fisica di Roma.

Finally the encouragement to start writing these notes from Unione Matematica Italiana and Carlo Pucci has been essential.

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