Errata-Corrige-Addenda-Variationes to the book “Foundations of fluid mechanics”, by G. Gallavotti

Note:
1. 2 means line 2 (from top of page);
2. -3 means line 3 from bottom of page

action to be taken for correction is typed in bold face

• p. 8, l. -1:
  \[ f(w) = e^{-\frac{m w^2}{2 \pi k_B T}} \left( \frac{m}{2\pi k_B T} \right)^{3/2} \]
  should be
  \[ f(w) = \left( \frac{m}{2\pi k_B T} \right)^{3/2} e^{-\frac{m w^2}{2 k_B T}} \]

• p. 8, l. -5: delete \( \tau = \lambda / \bar{v} \)

• p. 9, l. 4: delete recalling that

• p. 9, l. 5: \( \frac{d\bar{v}(z)}{dz} \) should be
  \( \frac{d\bar{v}(z)}{dz} \), by \( \bar{v} \) the mean velocity \( \bar{v} = (3 k_B T / m)^{1/2} \) and the free flight time by
  \( \tau = \lambda / \bar{v} \)

• p. 9, l. 14: and hence that should be
  so that, if the collision cross section is denoted \( \sigma \), it is

• p. 9, l. 6 in problem 1.1.4: \( kT \) should be \( k_B T \)

• p. 9, l. 5 in problem 1.1.5: deduce that \( \eta = \bar{\eta} \); should be
  deduce that \( \eta \) is the quantity studied in problem [1.1.4];

• p. 12, l. -1: per unit \( \Omega \) time delete spurious characters \( \Omega \)

• p. 13, l. 8 in probl. 1.1.16: production \( \Gamma \) should be production \( \Gamma \)
  (see problem 1.1.14)

• p. 17: before Sec. 1.2.3 add the following paragraph
  However one should remark that if the fluid is enclosed in a container \( \Omega \) the above equations might lead to results that might be physically unacceptable: for instance if the NS equations are subject to a boundary condition \( u = \bar{u} \) then the third equation shows that it will not be in general possible to fix the temperature at the boundary because the equation would imply that \( \partial_s > 0 \) on the boundary and \( s \) (hence \( T \)) will increase with time and could not be held fixed. Hence in presence of thermoconduction the above equations will be an acceptable model only in special cases; see Sec. 5 for a
treatment of the problem in presence of boundaries. The incompressible NS equations will therefore be acceptable only if the internal generation of heat can be completely neglected either because “small” or because it is assumed to be removed by some mechanism which is not described by the equations themselves or which is implemented through special boundary conditions, see Sect. 1.5.

- p. 18, l. 3: $\sigma(T) = \int c \, dT/T$, should be $\sigma(T) = \int c \, dT/T$ for some function $c = c(T)$,

- p. 18, l. 4 after (1.2.8): delete paragraph starting with The decoupling and replace it by:
The interpretation problems mentioned for the Euler and NS equations are still present. Furthermore the decoupling is often illusory because the equations for $s$ and $u$ might become incompatible with the boundary conditions.

- p. 18, l. 9 after (1.2.8): In addition should be In conclusion

- p. 19, l. 1 before (1.2.12): Hence should be Hence the condition (1.2.10) becomes

- p. 20, l. 2/3: (1.2.11) which, therefore, adds should be (1.2.11): hence the (1.2.10) adds

- p. 21, l. 2: conditions. should be conditions, apart from the problems with boundary conditions that we have only mentioned in Sect. 1.2.2 and 1.2.3, and which will be studied in some detail in a simple case in Sect. 1.5.

- p. 23, l. -3: $r \Delta u$ should be $\nu \Delta u$

- p. 26, l. -8: lowest order in $\varepsilon$. should be lowest order in $\varepsilon$ at least.

- p. 28, l. 5 in theorem 1: time $\tau_0$ should be time $\tau_0$, dimensionless by definition,

- p. 28, l. 5 after (1.3.11): $t < \tau_0 \varepsilon v_{\text{sound}}$ should be $t < \tau_0 \varepsilon v_{\text{sound}}$

- p. 28, l. -11: time $\tau_0$ should be time $\tau_0 \varepsilon v_{\text{sound}}$
• p. 29, eq. (1.1.13): Eq. (1.1.13) should be
\[ \frac{\nu}{\varepsilon \nu_{\text{sound}}} \equiv \nu_0 \ll 1 \] independently of \( \varepsilon \) \hspace{1cm} (1.3.13)

• p. 31, l. -11: gas is should be gas was

• p. 40, l. 2: is the value should be which is the value

• p. 40, l. 9: in prob. [1.4.7] kilometre should be kilometer

• p. 41: problem [1.4.10]
  ocean of depth \( h > 0 \), large enough.
  should be
  ocean of depth \( h > 0 \), large enough, and of density negligible with respect to that of the planet.

• p. 42: problem 1.4.14 at the end:
  per century,) should be
  per century, obviously showing the inadequacy of the model, see [MD00] for a theory of tides and despinning.)

• p. 42: problem 1.4.15: delete problem 1.4.15

• p. 44, l. 6 after (1.5.1): will be constants that should be will have values that

• p. 45, l. 2: add
 , aside from problems that might be generated by boundary conditions.

• p. 54, l. 2 after (1.6.5): : this is a form of the should be according to the

• p. 54, l. -10: consider vector fields should be consider divergenceless vector fields

• p. 55, l -12: displace here the figure 1.6.6 from the next page

• p. 57, l. 1 before (1.6.12): and a set \( \varphi \) of solutions should be and calling \( \varphi \) a solution

• p. 64, in formula (!): \[ \left( \frac{4}{\pi} \right)^p \] should be \( \left( \frac{4}{\pi} \right)^p \)
• p. 64/65: delete problems 1.6.17, 1.6.18, 1.6.19 and replace by

[1.6.17]: (Approximating an irrotational field with a gradient)

Let \( \gamma_n(x) = e^{(x^2 - n^2) - 1} c_n \) for \( |x| < 1/n \) and let \( \gamma_n = 0 \) otherwise. The constant \( c_n \) is such that \( \int \gamma_n \equiv 1 \), (\( c_n \propto n^3 \)). Let \( f \in L_2(\Omega) \) and \( \text{rot} f = 0 \)

in the sense that for each \( A \in C_0^\infty(\Omega) \) it is \( \int_\Omega A \cdot \text{rot} A = 0 \). Show, (with the

notations of the problem [1.6.16]) and if * denotes the convolution product

in the domain \( Q \) considered as a torus):

(i) \( \int_Q f^c \cdot \text{rot} A = 0 \) for \( A \in C_0^\infty(\cup_{k=1}^8 \Omega_k) \)

(ii) if \( n \) is large enough: \( \int_Q \gamma_n \ast f^c \cdot \text{rot} A^c \equiv 8 \int_\Omega \gamma_n \ast f^c \cdot \text{rot} A^c \equiv 0 \)

(large enough means that \( 1/n \) is smaller than the distance between \( \partial \Omega \) and the support of \( A \);

(iii) \( \text{rot} (\gamma_n \ast f^c) = \frac{\Omega}{2} \) in \( \Omega^{(n)}(\{ \frac{x}{n} \in \Omega_k, \text{d}(\frac{x}{n}, \partial \Omega_k) > \frac{1}{n} \}) \)

(iv) \( \lim_{n \to \infty} \gamma_n \ast f^c \equiv \frac{f}{2} \) in \( L_2(\Omega) \).

[1.6.18]: Within the context of problems [1.6.16], [1.6.17] suppose \( \Omega \) convex

and let \( \varphi_n(x) \) be defined for \( x \in \Omega_k^n \) in the interior of \( \cup_{k=1}^8 \Omega_k^{(n)} \) by

\[
\varphi_n(x) = \int_{\Omega_k^{(n)}} \frac{dy}{|\Omega_k^{(n)}|} - 1 ds (x - y) \cdot (\gamma_n \ast f^c)(y + s(x - y))
\]

Check that for \( x \in \cup_{k=1}^8 \Omega_k^{(n)} \)

\[
\partial_j \varphi_n(x) = \gamma_n \ast f^c(x), \quad \int_{\Omega_k^{(n)}} \frac{dy}{|\Omega_k^{(n)}|} \varphi_n(x) \leq C ||f||_{L_2(\Omega)}
\]

(Hint: The first relation is simply due to \( \text{rot} \gamma_n \ast f^c = 0 \) in \( \cup_{k=1}^8 \Omega_k^{(n)} \), c.f.r. problem [1.6.17]. The integral in the second can be bounded by

\[
L V \sqrt{3} \int_0^1 ds \int_{\Omega_k^{(n)}} \frac{dy}{|\Omega_k^{(n)}|} |(\gamma_n \ast f^c)(y + s(x - y))| \leq
\]

\[
\leq \frac{L V \sqrt{3}}{|\Omega_k^{(n)}|} \int_0^1 ds \int_{\{x - \frac{z}{n} < (1 - s)L\sqrt{3} \}} dxdz |(\gamma_n \ast f^c)(z)| \leq
\]

\[
\leq C_1 L \int_{\Omega_k^{(n)}} dxdz |(\gamma_n \ast f^c)(z)| \leq
\]

\[
\leq C_2 L^{5/2} ||\gamma_n \ast f^c||_{L_2(Q)} \leq C L^{5/2} ||f||_{L_2(\Omega)}
\]

(after changing variables \( y \to z = y + (x - y)s \) for suitable constants \( C_1, C_2, C \).)

[1.6.19] (Representing an irrotational field defined in a convex domain as a gradient) Let \( F_j^{(n)} \in L_2(\Omega) \) and \( \varphi_n \in \cup_{k=1}^8 \Omega_k^{(n)} \) be chosen, in the context

of problems [1.3.17], [1.3.18], so that

\[
\partial_j F_j^{(n)} = \gamma_n \ast f^c \text{ in } L_2(Q), \quad ||\varphi_n||_{L_2(\Omega_k^{(n)})} \leq C ||f||_{L_2(Q)}
\]
Check first that the bounds obtained in problem [1.6.18] imply that in $\Omega^{(n)}_k$ it is $F_j^{(n)}(x) = \partial_j \varphi_n(x) + c_k^j$ with the constant $c_k^j$ bounded by $C\|f\|_{L_2(\Omega)}$.

Then extend $\varphi_n$ to $Q$ by setting it equal to 0 outside $\cup_{k=1}^8 \Omega^{(n)}_k$ and let $\varphi = \lim \varphi_n$ be a weak limit in $L_2(\Omega)$ of $\varphi_n$ on a subsequence $n_q \to \infty$. Show that $\varphi$ admits generalized first derivatives and $\partial_i \varphi \equiv f$. (Hint: One remarks that if $b \in C_0^\infty(\Omega)$ it is $\int_{\Omega} \partial_j b \varphi_n = \lim_{n \to \infty} \int_{\Omega} \partial_j b \varphi_n = -\int_{\Omega} b \partial_j \varphi_n = -\int_{\Omega} b f_j$ so that $|\int \varphi \partial_j b| \leq \|f\|_{L_2(\Omega)} \|b\|_{L_2(\Omega)}$.)

- p. 65, l-8: replace text from Continue to flux generated by

Continue the field in such a way that its flux lines exit the right face of the cylinder (where $x = L$ and the other two coordinates are $y, z$, say) and reenter from the left side (where $x = 0$) at the point corresponding to the same coordinates $y, z$: this is done by defining $u$ outside the cylinder so that the flux thus generated

- line 2 after formula (1.7.2) line 2: fluid should be fluid

- formula (1.7.5) line 2: $(d\xi + t \partial_t u(\xi, 0)) \cdot d\xi$ should be $(d\xi + t \partial_t u(\xi, 0)) \cdot d\xi$

- p. 70, l. -7: An incompressible should be A smooth incompressible

- p. 71, l. 7: a velocity should be a smooth velocity

- p. 72, l. 13/14: as in the case should be as it is possible in the case

- p. 73, l. 7: which make should be which makes

- p. 73, l. 6 after (1.7.26): $\delta F/\partial f(x)$ should be $\delta F/\partial f(x)$

- p. 75, l. 1 before (1.7.31): Delete text from Hence through l. 1 before (1.7.36) and replace by

Hence, denoting $\zeta = \partial_t \zeta, \zeta' = \partial_x \zeta$, we obtain the boundary condition $-\partial_x u_x + u_z = \partial_t \zeta$ in $x, \zeta(x, t)$; the latter relation has to be coupled with the condition that the pressure be constant, e.g. $p = 0$, at the free boundary. If $n(x) \equiv (-\zeta'(x), 1)/(1 + \zeta'(x)^2)^{1/2}$ is the external normal to the set of the $(x, z)$ with $z < \zeta(x)$, the two conditions can be written as

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\[
(u \cdot \vec{\partial} \varphi)(x, \zeta(x)) = \frac{\partial \zeta(x)}{(1 + \zeta'(x)^2)^{1/2}}
\]
\[
\partial \varphi + g \zeta + \frac{1}{2}(\partial \varphi)^2 = 0
\] (1.7.31)

respectively, c.f.r. (1.7.7). The relations show that the two functions \((\zeta(x), \dot{\zeta}(x))\) uniquely determine the velocity field as well as their values at time \(t + dt\).

In fact given \(\zeta, \dot{\zeta}\) we can compute the velocity field \(u = \vec{\partial} \varphi\) by determining \(\varphi\) as the harmonic function satisfying \(\Delta \varphi = 0\) in \(D = \{(x, z) | x \in [0, L], z \in (-\infty, \zeta(x))\}\) with boundary condition \(\partial_{n} \varphi\) given by the first of (1.7.31): a Neumann problem; furthermore from the second relation we can compute \(\partial_{t} \varphi(x, z)\) by solving a Dirichlet problem in \(D\) with boundary condition \((\partial_{t} \varphi)(x, \zeta(x))\). The latter field can be used to compute \(\ddot{\zeta}(x)\) by differentiating the boundary condition rewritten in the form \(\partial_{t} \zeta = -\partial_{x} \zeta u_{x} + u_{z}\).

We can now put the equations into Hamiltonian form. We expect that, the motion being “ideal”, the equations of motion must follow from a naive application of the action principle. We shall restrict the analysis to motions that are horizontally periodic over a length \(L\), for simplicity. Since we have seen that the motions are naturally described in the coordinates \((\zeta(x), \dot{\zeta}(x))\) we consider the action functional of \(\zeta, \dot{\zeta}\) as the difference between the kinetic energy of the fluid and its potential energy in the gravity field.

Hence the domain \(D\) in which we shall consider the fluid is \(D = \{(x, z) | x \in [0, L], z \in (-\infty, \zeta(x))\}\). In \(D\) the velocity field will be \(u = \vec{\partial} \varphi\) and its time derivative will be \(\dot{\varphi}\) with \(\varphi\) harmonic, solutions of

\[
\Delta \varphi = 0 \quad \text{in } D, \quad u \cdot \vec{\partial} \varphi = \frac{\dot{\zeta}}{(1 + \zeta'(x)^2)^{1/2}} \quad \text{in } \partial D
\] (1.7.32)

The potential energy will obviously be infinite, being \(\int_{0}^{L} \int_{-\infty}^{\zeta(x)} \rho g z \ dx \ dz\). However what counts are the energy variations which are formally the same as those of \(\rho \int_{m}^{\zeta(x)} dx \int_{-\infty}^{\zeta(x)} \rho g z \ dx \ dz\) where \(m\) is any quantity less than the minimum of \(\zeta(x)\).

The kinetic energy will be \(\frac{\rho}{2} \int_{0}^{L} dx \int_{-\infty}^{\zeta(x)} (\partial \varphi)^2 \ dx \ dz\). Thus the action of the system, i.e. the difference between kinetic and potential energy is, if \(ds \equiv (1 + \zeta'(x)^2)^{1/2} \ dx\) and if we introduce the auxiliary function \(\psi(x) \equiv \varphi(x, \zeta(x))\),

\[
\mathcal{L} = \frac{\rho}{2} \int_{0}^{L} \int_{-\infty}^{\zeta(x)} (\partial \varphi)^2 \ dx \ dz - \frac{\rho}{2} \int_{0}^{L} g \zeta(x)^2 \ dx = \frac{\rho}{2} \int_{0}^{L} (\psi(x)) (\dot{\zeta}(x)) \ dx \ dz - g \zeta(x)^2 \ dx
\] (1.7.33)

after integrating by parts the first integral, defining if \(\psi(x) \equiv \varphi(x, \zeta(x))\) and making use of the first relation (1.7.31), (1.7.32).
The linearity of (1.7.32) shows that \( \psi \) is linear in \( \dot{\zeta}(x) \):

\[ \psi = M \dot{\zeta}/(1 + (\zeta')^2)^{1/2} \]

for some linear operator \( M \) (which depends on \( \zeta(x) \), i.e. on the shape of \( D \)).

In general if \( \varphi_1, \varphi_2 \) are two harmonic functions in \( D \) with respective “Dirichlet value” \( \varphi_1, \varphi_2 \) on the boundary, \( \partial D \), and respective “Neumann values” on the same boundary \( \sigma_1 = \mathbf{n} \cdot \frac{\partial \varphi_1}{\partial x}, \sigma_2 = \mathbf{n} \cdot \frac{\partial \varphi_2}{\partial x} \) then, (on the boundary) by definition, \( \varphi_i = M \sigma_i \) for a suitable linear operator \( M \) and

\[ \int_{\partial D} ds \left( (M\sigma_1) \sigma_2 - (M\sigma_2) \sigma_1 \right) = 0 \]

so that the operator \( M \) is symmetric on \( L_2(ds, [0, L]) \). Note that the operator \( M \) transforms, by definition and by (1.7.34), a Neumann boundary datum for a harmonic function into the corresponding Dirichlet datum. Hence

\[ L(x) = \rho \int_0^L \frac{G(\dot{\zeta}) ds}{(1 + (\zeta')^2)^{1/2}} - g \zeta^2 dx \]

(1.7.34)

Therefore we can write the equations of motion in terms of the Hamiltonian corresponding to the Lagrangian \( L \). Note that \( \delta L/\delta \dot{\zeta}(x) = M(\ddot{\zeta}(x)/(1 + (\zeta')^2)^{1/2}) \), where we denote by \( \delta/\delta \varphi(x) \) a generic functional derivative and we recall that \( (1 + (\zeta')^2)^{-1/2} ds \equiv dx \): therefore we can define the variable \( \pi(x) \) conjugate to \( \zeta(x) \)

\[ \pi(x) = \rho \frac{\dot{\zeta}(x)}{(1 + (\zeta')^2)^{1/2}} = \rho \psi(x) \]

and the Hamiltonian in the conjugate variables \( \pi, \zeta \) is

\[ H = \frac{1}{2} \int_0^L \left( \frac{G(\dot{\zeta})}{\pi(x)} + \rho \dot{\zeta}(x)^2 \right) dx \]

(1.7.35)

where if \( \Gamma^D \) is the operator that solves the Dirichlet problem in \( D \) we set

\[ G(\zeta) = \rho \left( - \partial_z \zeta(x) \partial_x (\Gamma^D \pi(x,z)) + \partial_x (\Gamma^D \pi(x,z)) \right) \]

The check that the equations of motion generated by (1.7.35) are correct, i.e. coincide with (1.7.31) proceeds as follows. The first Hamilton equation is just \( \dot{\zeta} = \frac{1}{\rho} G\pi \) which coincides with the first of (1.7.31). More interesting is the computation of the functional derivative \( -\delta H/\delta \zeta(x) \). It is

- p. 77, l. 1/2 after (1.7.36): deduced from should be studied by remarking that by its definition \( (\Gamma^D \pi(x,z)|_{z=\zeta(x)} \equiv \pi(x) \) hence near \( z = 0 \) we get

- p. 85, l. -9: significant should be most significant

- p. 87, l. 1: Since the level lines of \( \sigma_\varepsilon(\xi) \) are, should be The level lines of \( \sigma_\varepsilon(\xi) \) will be,

- p. 87, l. 2: the gradient should be and the gradient
p. 87, l. 16: its divergence should be the divergence

p. 87, l. 17: tends should be would tend

p. 88: in formulae (2.1.17) and (2.1.19): \( \sum_{h=1}^{k} \) should be \( \sum_{h=0}^{k-1} \)

p. 89, l. 3 after (2.1.9): \( \varepsilon \to 0 \). should be \( \varepsilon \to 0 \) (see the last term in the second equation).

p. 89, l. -10: \( (\sigma_{\varepsilon}) \) should be \( (\overline{\partial} \sigma_{\varepsilon}) \)

p. 93, l. 1 in item (i) of prop. 2: \( \tau, b, \eta > 0 \). should be \( \tau, b, \eta > 0 \) (implying, among other things, that \( T_{0} \) is analytic entire in \( x \))

p. 93, l. 1 in item (iii) of prop. 2: if should be unless

p. 93, l. 2 in item (i) of prop. 2: the solution of should be the "usual", well known, solution \( T(x, t) \) of

p. 93, l. 1-2 in item (iii) of prop. 2: delete (implying, among other things, that \( T_{0} \) is analytic entire in \( x \)) to be inserted in item (i)

p. 93, l. 2 in item (iii) of prop. 2: is in general should be is not, in general

p. 93, l. 1 after (2.1.23): \( \omega = 2 \pi n \) should be \( \omega = n \)

p. 93, l. -5: \( T \) should be \( T \) two replacements

p. 93, l. -5 -4: the solution of (2.1.12). should be the "usual" solution of (2.1.12) with Fourier transform given by the r.h.s. of (2.1.24).

p. 93, l. -1: \([-a, a] \). should be \([-a, a] \), hence it cannot be analytic for \( t > 0 \) (while \( T(x, t) \) is analytic).

p. 94, l. -13: a well defined solution and keeping should be a solution
which is well defined and keeps

- p. 94, l. -9: equal to 0 should be equal to 0 or to each other

- p. 94, l. -4: are likely to be solutions should be are, in some sense, solutions

- p. 94: change the footnote 3 to 3 I do not know a reference for detailed analysis of this property.

- p. 95, l. 2: ±a). should be ±a, to say the least).

- p. 95, l. 5 and l. -1: [−π, π]. should be [−π, π] and for all times \( t \geq 0 \).

- p. 95, l. 10 in remark (5): Euler equations or equations can “hide” major existence and Navier Stokes equations. and lead should be Euler or Navier–Stokes equations and lead

- p. 95, l. -1: [−π, π]. should be [−π, π] and \( t \geq 0 \).

- p. 96, l. 4: (see [1.1.5]) should be (see problem [1.1.5])

- p. 96, l. -1: labelled should be labeled

- p. 97, l. 2: delete Note that the second bound is better than the first if \( \varepsilon n \) is large.

- p. 98, probl. 2.1.8: replace problem 2.1.8 by

2.1.8 Consider, in the case \( d = 2 \), a solenoidal nonconservative force \( g \in C^\infty(\Omega) \) and let \( \gamma = \partial \Omega \) be such that \( I = \int_\gamma g \cdot dx \neq 0 \). Show that in this case (2.1.9) is not, in general, soluble if the initial datum is \( u_0 = 0 \). Find an example. (Hint: Let \( \Omega \) be a disk of radius \( r \) and let \( g(x) \overset{\text{def}}{=} \omega \wedge x \) with \( \omega = \omega_0 \) orthogonal to the disk. Then the uniqueness of the solutions for the Neumann problem implies that \( p = \text{const} \) hence \( \frac{\partial p}{\partial n} = g = 0 \) but \( I = 2\pi r^2 \omega \).

- p. 98, l. -13: \( \equiv 2 \) should be \( \equiv ( \)

- p. 100, l. 10: avoid the condition that should be exclude that
• p. 100-101: in formulae (2.2.7),(2.2.8),(2.2.10),(2.2.11),(2.2.12): \( \prod \) should be \( \Pi \) i.e. capital greek \( \pi \)

• p. 101, l. -10: “ultraviolet” components also called the “short wave” components should be “short wave” components, also called the “ultraviolet” components,

• p. 101, in (2.2.13): \( \sum_{n=1}^{n} \) should be \( \sum_{m=0}^{n-1} \)

• p. 102, l. 10: should should be should

• p. 107, l. 1: \(-k_{1}^{4}\nu\omega_{k_{1}}\) should be \(-k_{1}^{2}\nu\omega_{k_{1}}\)

• p. 107, l. 10: \(\omega_{k_{1}}\)'s thought should be \(\omega_{k_{1}}\)'s, thought

• p. 107, l. 11: gyroscope must should be gyroscope, must

• p. 110, l. 5: solution should be solution of NS

• p. 111, problem (2.2.3): the first line should be the first line of the next prob. (2.2.4), while the first line of probl. (2.2.3) should be (Weak solutions ambiguities)

• p. 111, l. 6. in prob. (2.2.4): (cf. (2.2.15) by the definition of \(D\)) should be (by the definition of \(D\), cf. (2.2.15))

• p. 111, l. 6. in prob. (2.2.5): \(dx_{\mu} \) should be \(dx_{\mu} \) and also \(dx \) should be \(dx \)

• p. 111, l. 9. in prob. (2.2.5): \(f_{Q}(\partial u)^{2} \) should be \(f_{Q}(\partial u)^{2}/f_{Q}u^{2} \)

• p. 111, l. 10. in prob. (2.2.5): \(C_{\infty}^{0}(\Omega_{\text{mega}}) \) should be \(C_{\infty}^{0}(\Omega) \)

• p. 112, in formula (2.2.39): \(D(u_{n_{i}}) \) should be \(D(\tilde{u}_{n_{i}}) \)
• p. 112, l. -5: \(|u_n - u|_{n \to \infty} = 0\) and \(D(u_n - u)_{n \to \infty} = 0\) should be \(|u_n - u|_{n,m \to \infty} = 0\) and \(D(u_n - u)_{n,m \to \infty} = 0\).

• p. 114-115-116-117: replace problems 2.2.21, 2.2.22, 2.2.23 2.2.24, 2.2.25, 2.2.26 by

[2.2.19] (Traces) Let \(\Omega\) be a domain with a bounded smooth manifold as boundary \(\partial \Omega\): i.e. such that \(\partial \Omega\) is covered by a finite number \(N\) of small surface elements each of which can be regarded as a graph over a disk \(\delta\), tangent to \(\partial \Omega\) at its center \(\xi_0 \in \partial \Omega\), so that the parametric equations of \(\sigma_i\) can be written \(z = z_i(\xi), \xi \in \delta\) and the points \(\xi\) of \(\Omega\) close enough to \(\sigma_i\), can be parameterized by \(\xi = (\xi, z_i(\xi) + z)\) with \(\xi \in \delta, z \geq 0\). Note that if \(\Omega\) has the above properties also the homothetic domains \(\rho \Omega\) with \(\rho \geq 1\) have the same properties and the \(\sigma_i\) can be so chosen that \(\text{diam}(\sigma_i) = cL\) if \(L\) is the diameter of \(\Omega\) and \(N, c\) are the same for all domains \(\rho \Omega\) with \(\rho \geq 1\).

Let \(f \in C^\infty(\Omega)\) and define \(\partial^2_{\xi} \equiv \sum_{\delta^2_{\xi}} - \sum_{\delta^2_{\xi}}\) and

\[
||f||_{W^2(\Omega)}^2 = \sum_{j=0}^n L^{2j-d} \sum_{|\alpha| < j} \int_\Omega |D^\alpha f(\xi)|^2 d\xi,
\]

\[
||f||_{W^n(\sigma_i)} = ||f_\delta||_{W^n(\delta_i)}, \quad ||f||_{W^n(\partial \Omega)} = \max_i ||f_\delta||_{W^n(\delta_i)}
\]

where \(f_\delta(\xi) = f(\xi, z_\delta(\xi))\). Check that problem [2.2.18] implies

\[
||f||_{W^{n-1}(\partial \Omega)} \leq \Gamma ||f||_{W^n(\Omega)}, \quad n \geq 1
\]

and the constant \(\Gamma\) can be taken to be the same for all domains of the form \(\rho \Omega\), \(\rho \geq 1\).

[2.2.20] (A “trace” theorem) Given a scalar function \(f \in L_2(\Omega)\) suppose that it admits generalized derivative up to the order \(n\) included, with \(n\) even. Hence \((-\Delta)^{n/2}\) exists in a generalized sense, because \(|(f, (-\Delta)^{n/2} g)| \leq C||g||_2\) for \(g \in C^0_0(\Omega)\) and for a suitable \(C, c,f.r. \S 1.6, (1.6.19)\) and problem [2.2.1] above. Suppose \(n > d/2\).

Show that \(f\) is continuous in every point in the interior of \(\Omega\), together with its first \(j\) derivatives if \(n - d/2 > j \geq 0\). Find an analogous property for \(n\) odd.

(Hint: Let \(Q_\delta\) be a cube entirely contained in \(\Omega\) and let \(\chi \in C^\infty_0(Q_\delta)\). The function \(\chi f \equiv f_\delta\) thought of as an element of \(L_2(Q_\delta)\) admits generalized derivatives of order \(\leq n\) and \((-\Delta)^{n/2} f_\delta\) exists in a generalized sense (this is clear if \(n/2\) is an integer because \(\chi \partial^p g = \sum_{j=0}^p \partial^j(\chi^{(j)} g)\) where \(\chi^{(j)}\) are suitable functions in \(C^0_0(Q_\delta)\)).

Hence thinking of \(g \in C^\infty_0(Q_\delta)\) as a periodic function in \(Q_\delta\) we see that there is a constant \(C_Q\) such that the relation \(|(f_\chi, (-\Delta)^{n/2} g)| \leq C_Q \cdot ||g||_{L_2(Q_\delta)}|\) holds for each \(g \in C^\infty_0(Q_\delta)\), and therefore it must hold for each periodic \(C^\infty_0(Q_\delta)\)–function \(g\). Then, if \(f_\delta(K)\) is the Fourier transform of \(f_\delta\) as element of \(L_2(Q_\delta)\) (so that \(K = 2\pi \varepsilon^{-1} m\) with \(m\) an integer components vector), we
get \( \varepsilon^d \sum \| \hat{f}_x(k)^2 \| k^{2n} \leq C_{Q_x}^2 \). Hence, setting \( n = \frac{d}{2} + \eta \) with \( 1 > \eta > 0 \), we see that the Fourier series of \( \partial^j f_x \) is bounded above by the series

\[
\sum_k |k|^j |\hat{f}_x(k)| = \sum_k |k|^{j+\eta+d/2} |\hat{f}_x(k)| |k|^{-\eta-d/2} \leq \\
\leq \left( \sum_k |k|^{2n} |\hat{f}_x(k)|^2 \right)^{1/2} \left( \sum_k |k|^{-2n-\eta} \right)^{1/2} \leq \\
\leq C_{Q_x} \varepsilon^{-d/2} \left( \frac{\varepsilon}{2\pi} \right)^{\eta+d/2} \left( \sum_m |m|^{-d-2\eta} \right)^{1/2} \equiv \Gamma \varepsilon^{n-j-d/2} C_{Q_x}
\]

hence \( f \) has \( j \) continuous derivatives in \( Q_x \). If \( n \) is odd one can say, by definition that \( (-\Delta)^{n/2} f \) exists if it is \( \langle f, \partial^j g \rangle \leq C \| g \|_2 \) for each derivative of order \( j \leq n \) and for each \( g \in H^2_0(\Omega) \); then the discussion is entirely parallel to the one in the \( n \) even case.)

**[2.2.21](An auxiliary trace theorem)** Let \( W^n(\Omega) \) be the space of the functions \( f \in L_2(\Omega) \) with generalized derivatives of order \( \leq n \) and define. Show that the method proposed for the solution of [2.2.19] implies that, if \( n > j + d/2 \) and \( d(x, \partial \Omega) \) denote the distance of \( x \) from \( \partial \Omega \), then

\[
L^j |\partial^{\alpha} f(x)| \leq \left( \frac{d(x, \partial \Omega)}{L} \right)^{-j-d/2} \Gamma \| f \|_{W^n(\Omega)}, \quad |\alpha| = j
\]

and \( \Gamma \) can be chosen to be independent of \( \Omega \). (*Hint: Let \( Q_1 \) be the unit cube. Let \( \chi_1 \in C_0^\infty(Q_1) \) be a function identically equal to 1 in the vicinity of the center of \( Q_1 \). Let \( \chi_\varepsilon(x) = \chi_1(\varepsilon^{-1} x) \) and show that the constant \( C_{\varepsilon_0} \) considered in the estimates of problem [2.2.19] can be taken equal to \( \gamma L^{d/2}\varepsilon^{-n} \| f \|_{W^n(\Omega)} \), with \( \gamma \) independent from \( \Omega \). Then choose \( \varepsilon = d(x, \partial \Omega) \).*)

**[2.2.22] (Trace theorem)** Infer from problem [2.2.19], [2.2.21] that if \( j < n - 1 - d/2 \) there is a constant \( \Gamma \) such that \( \partial^{\alpha} f(x), |\alpha| = j \) with \( x \in \partial \Omega \) is continuous and

\[
L^j |\partial^{\alpha} f(x)| \leq \Gamma \| f \|_{W^n(\Omega)}, \quad |\alpha| = j
\]

(*Hint: a point \( x \in \partial \Omega \) will be in some \( \sigma_i \), c.f.r. problem [2.2.19], and at a distance \( O(1) \) from its boundary. Then apply problem [2.2.21].*) With a little extra effort one can obtain the “same” result under the weaker condition \( j < n - d/2 \).

- p. 118, l. 1: *Dirichlet form* should be
  *Dirichlet form: scalar case*

- p. 118, l. 2: \( f_n \in X^0_{rot}(\Omega) \) should be \( \| f_n \|_2 = 1 \)
• p. 118, l. 5: delete the hint and replace it by (Hint: For each \( g \in C_0^\infty(\Omega) \) we get, setting \((f, g)_D \equiv \int_\Omega \mathbf{\hat{u}} f \cdot \mathbf{\hat{u}} g \, d\xi\), that
\[
|(f_n, g)_D - \lambda_0^2(f_n, g)| \xrightarrow{n \to \infty} 0 \Rightarrow |(f_0, (\Delta - \lambda_0^2)g)| = 0
\]

Hence \(|(f_0, -\Delta g)| \leq \lambda_0^2||g||_2\) and \(f_0\) has a Laplacian in a generalized sense, by definition of generalized derivative, and \(\Delta f_0 = \lambda_0^2 f_0\).)

• p. 119, l. 1 in problem [2.2.33]: should be (Estimates on the large order eigenvalues and eigenfunctions of the scalar Dirichlet form) scalar

• p. 119, l. 1 before problem [2.2.34]: add Lower bounds on \(\lambda_j(\Omega)\) can be obtained by the minimax principle by enclosing \(\Omega\) in a cube \(Q' \supset Q \supset \Omega\).

• p. 119, l. 1 in problem [2.2.34]: should be (estimates on the large order eigenvalues and eigenfunctions of the scalar Dirichlet form)

• p. 122, l. -11: make not should be not make

• p. 124, l. -10: \(\omega = \omega_1\) should be \(\omega \overset{\text{def}}{=} \omega_1\)

• p. 124, l. -11: add
The distance \(\sqrt{\Delta}\) has to be \(>0\) as we shall exclude initial data in which a pair of vortices occupy the same point.

• p. 125, l. -19/-18: replace the sentence by: the circulation at \(\infty\) has indeed the value \(\omega\).

• p. 126, l. -6: \(\log(p'^2 + q'^2)\) should be \(\log(p'^2 + q'^2)\)

• p. 127, l. 2 before prob. 2.3.9: linear and should be linear, and

• p. 128, l. 7: \(\frac{2}{\pi} \frac{(m \cdot \xi)^2 \xi^2}{(m^2 \xi^2)^2}\) should be \(\frac{1}{2} \frac{(m \cdot \xi)^2 \xi^2}{(m^2 \xi^2)^2}\)

• p. 130, l. 1 before eq. (2.4.8): the Biot–Savart law replace by a formula often called Biot–Savart formula because it says that the velocity field of the vorticity field is the “magnetic field” of an electric current of
intensity $\Gamma$ circulating on the filament as computed from the Biot-Savart law (units aside, of course)

- p. 132, l. -2: Frenets’ should be Frenet’s

- p. 134, l. 13: is not obviously defined should be cannot be, a priori, considered defined

- p. 136, l.2/3 of (2.4.27) : $(D|\alpha|)$ should be $D|\alpha|$

- p. 137, l. -13: $\omega^I$ should be $\omega^A$

- p. 139, l. 2 after (2.4.33): $s \rightarrow \xi(s) = (s, \eta s)$ should be $s \rightarrow \xi(s) = (s, \eta s), 0 \leq s \leq q_n,$

- p. 139, l. 4 after (2.4.33): (c.f.r. [Ga83] should be (cf. problem [5.1.7] below)

- p. 139, l. 9/10 after (2.4.33): where $dh$ is the size should be where $d\ell$ is the size

- p. 139, in eq. (2.4.35): $l_n dh d\ell$ should be $l\nu dh d\ell$

- p. 140, l. 6: then $u$ results integrable. should be then the integral expressing the value of the field $u$ is convergent.

- p. 140, l. 3 in prob. 2.4.2: labelled should be labeled

- p. 142, l. 1 before Biblio.: exponent 1/2 should be any exponent $< 1/2$

- p. 142, in Biblio.: shift the sentence
The Gaussian process defined in [2.4.4] and discussed in problems following [2.4.4] has periodic sample paths: it differs therefore from the usual brownian motion. However the difference is quite trivial, see [IM65] p.21, problem 3. to problem 2.4.4 before the hint and set it in italics

- p. 143: In Sec. 3.1 from beginning to problems excluded globally replace first $G$ with $V$ and then $\Phi$ with $G
There is $B_0 > 0$ such that if $\xi_0 \overset{\text{def}}{=} k_0 \xi \leq 1, V_c \overset{\text{def}}{=} \nu L^{-1}, T_c \overset{\text{def}}{=} L^2 \nu^{-1}$ and $V'_0 \overset{\text{def}}{=} B_0 (V + GT_c) \xi_0^{-d-1}$ then (3.1.2), admits a solution $u(x, t)$ analytic in $x$ and $t$ with

$$|\gamma_k(t)| \leq V_0 e^{-\xi_0 k^2 / 2k}, \quad \text{for} \quad 0 \leq t \leq t_0 \overset{\text{def}}{=} T_c \left( \frac{V_c}{V_0} \right)^2$$

(3.1.3)

and the solution is the unique one enjoying the above properties in the time interval $[0, t_0]$.

\[ p. 143, l. -1: \text{cf. [3.1.0]} \quad \text{should be \ cf. Problem [3.1.0]} \]

\[ p. 143, l. -4: \text{modify text from There to end of proposition as} \]

\[ p. 144, l. 14: 4 > \frac{3}{2} + 1 \quad \text{should be} \quad 4 > \frac{3}{2} + 2 \]

\[ p. 144, l. 14: [2.2.20] \quad \text{should be \ Problem [2.2.22]} \]

\[ p. 144, \text{in eq. (3.1.4):} -\nu \int_\Omega \delta u^2 \, dx \quad \text{should be} \quad -\nu \int_\Omega (\delta \bar{u})^2 \, dx \]

\[ p. 145, l. 1: (3.1.6) \quad \text{should be \ (3.1.5)} \]

\[ p. 145, l. 1 \text{after (3.1.6):} \varphi_k \text{ of} \quad \text{should be \ } \Pi_k \hat{g} \text{ of} \]

\[ p. 145, \text{in (3.1.7),(3.1.8),(3.1.9),(3.1.10),(3.1.12):} e^{-\nu k^2(t - \tau)} \quad \text{should be} \quad e^{-\nu k^2(t - \tau)} \text{ (repeated many times)} \]

\[ p. 150, l. 6: n \text{ nodes} \quad \text{should be \ } m \text{ nodes} \]

\[ p. 150, l. 7: n + 1 \text{ final} \quad \text{should be \ } m + 1 \text{ final} \]

\[ p. 150, l. 7: n - 1 \text{ internal} \quad \text{should be \ } m - 1 \text{ internal} \]

\[ p. 150, l. 1 \text{ in eq. (3.1.13):} n + 1 \quad \text{should be \ } m + 1 \]

\[ p. 150, l. 2 \text{ before eq. (3.1.14):} \text{by enlarging} \quad \text{should be \ enlarging to} \quad [0, t] \]

\[ p. 150, \text{eq. (3.1.14):} \text{All pairs} \quad kk \quad \text{should be} \quad k \]

\[ p. 150, \text{in eq. (3.1.15):} 2^{4n} \quad \text{should be} \quad 2^{4m} \]
• p. 151, l. 9:
\[\nu \text{ and } (16B\nabla_0\xi_0^{-d-1})^{-2} \text{ should be } \nu (16B\nabla_0\xi_0^{-d-1})^{-2}\]

• p. 151, l. 8 after (3.1.17): enough time. **should be**

enough time: this is the case not only for the “normal viscosity” (friction term \(\nu \Delta u\)) but also for the ipoviscous NS with friction term \(-\nu|\Delta|^\alpha\) with \(1 > \alpha > \frac{1}{2}\) and (of course) for the hyperviscous NS with \(\alpha > 1\).

• p. 151, l. 8 after (3.1.17): **add entries in the subject index pointing here with the following four items**

ipoviscous NS, hyperviscous NS, ipoviscosity, hyperviscosity
• p. 152, l. 4 in prob. (3.1.2): \[ \equiv \int u \cdot \partial w^2 \, dx \] should be \[ \equiv \frac{1}{2} \int u \cdot \partial w^2 \, dx \]

• p. 152, l. 6 in prob. (3.1.3): \[ [2.2.20] \] should be \[ [2.2.22] \]

• p. 152, l. 3 before prob. 3.1.4: \[ [2.2.20] \] should be \[ [2.2.22] \]

• p. 153, l. 2: \[ [3.1.2] \] should be \[ [3.1.3] \]

• p. 154, l. 8. in prob 3.1.10: \( p > 0 \) should be \( p > 2d \)

• p. 154, l. -8: quantities. should be quantities: rather it is estimated in the shorter interval \([0, \tau_{h+p}(R)]\).

• p. 154, l. 5 in prob. 3.1.9: add in the subject index the item constructive algorithm pointing at l. 5

• p. 155, l. 2/4: the sentence after points out in l. 2 should be points out that although by abstract arguments we can show existence and even some \textit{a priori} bound on \( ||w||_{W^1} \), in terms of \( t \) and \( \hat{w}_0 \), nevertheless \textit{a constructive estimate} for \( \tilde{w}^N \) is not known.

• p. 155, l. 8.: delete , as in [3.1.6],

• p. 155, l. 8.: (whose use is strongly not recommended to everybody) should be (whose use we strongly disrecommend)

• p. 155, l. -4: delete of \( \hat{u} \) solving the Euler equations and

• p. 155, l. 4 in prob. 3.1.11: \( \nu = 0 \) should be \( \nu = 0, g = \hat{u} \)

• p. 155, l. 4 in prob. 3.1.11: (2.3.3) should be (2.3.3) with \( \nu, \gamma = 0 \)

• p. 156, l. -3: The second one, \( n = \hat{0} \) term, is obtained should be The \( n = \hat{0} \) contribution to the second estimate

• p. 157, prob. 3.1.16: Replace the entire text of problem by [3.1.16]: \textit{(Continuous dependence of the flow lines from the vorticity field)} There exists \( C_0 \) such that, defining \( M_0 = ||\omega||_0 \) and \( \delta \overset{\text{def}}{=} e^{-C_0 M_0 T} \) for \( T > 0 \), then the currents generated by the two fields \( \tilde{w}^\zeta, \tilde{w}^{\zeta'} \) with \( \zeta, \zeta' \in \mathcal{M}_0 \) are
such that for all \( x \in \Omega \) and \( s, t \in [0, T] \)
\[
|U_{s,t}^\zeta(x) - U_{s,t}^{\zeta'}(x)| \leq C_0 M_0 T \left(||\zeta - \zeta'||_{0M_0^{-1}}\right)^\delta L
\]

Hence the \( R \) can be thought of as defined on the closure \( \overline{M}_0 \) of \( M_0 \) with respect to the metric of the uniform convergence and it can be extended to all continuous vector fields \( \zeta \), without any differentiability property, verifying the (2), (3), (4) of problem [3.1.11]. (Hint: Setting \( \rho_s = L^{-1}(U_{s,t}^\zeta(x) - U_{s,t}^{\zeta'}(x)) \), note that \( \rho_t = 0 \); furthermore from problem [3.1.15], by subtracting and adding \( v_{\zeta}(U_{s,t}^{\zeta'}(x), s) \), there exist constants \( C_1, C_2, C_0 \):
\[
|\partial_s \rho_s| = |L^{-1}[\zeta(U_{s,t}^\zeta(x), s) - \zeta(U_{s,t}^{\zeta'}(x), s)]| \leq C_1|\rho_s| \log_+ |\rho_s|^{-1} + L^{-1}|\zeta(U_{s,t}^\zeta(x), s) - \zeta(U_{s,t}^{\zeta'}(x), s)| \leq C_1|\rho_s| \log_+ |\rho_s|^{-1} + C_2||\zeta - \zeta'||_0 \leq C_0(|\rho_s| \log_+ |\rho_s|^{-1} + ||\zeta - \zeta'||_0)
\]
Since \( 0 \leq s \leq T \), by integration it follows that \( |\rho_s| \leq R \) if \( R \) is
\[
\int_0^R \frac{d\rho}{|\rho_0\log_+ \rho^{-1} + ||\zeta - \zeta'||_0M_0^{-1}} = C_0 M_0 T
\]
which means that \( R \) can be taken \( R \leq K(M_0T)(||\zeta - \zeta'||_{0M_0^{-1}})^\delta \) with \( K(M_0T) \) a continuous increasing function and \( \delta = \exp -M_0C_0T \).

- p. 157, l. 7 in prob. 3.1.16: uniform so convergence it can should be uniform convergence and it can
- p. 167, l. -1: (2.2.20) should be Problem [2.2.20]
- p. 167, l. -1: \( \partial T \) should be \( \partial \Omega \)
- p. 168, l. 4: min should be max
- p. 169, l. 1 after (3.2.31): \( \gamma(t) \) should be \( \gamma^\infty(t) \)
- p. 169, l. 4 after (3.2.31): \( (vt)^{-1} \) should be \( (k^2_0 + (vt)^{-1}) \)
- p. 172, l. 4 in remarks: [3.1.5] should be [3.1.6]
- p. 173, l. -1: \( |k_1| \) in the summation should be \( |k_1| \) or \( |k_2| \)
- p. 175, l. -7: reularity should be regularity
• p. 176, l. -8: does not only consist should be not only consists

• p. 176, l. -6: and therefore should be so that

• p. 176, l. -5: Indeed it is conceivable should be

It also consists in the possibility

• p. 177, in formula (3.3.3): \(+i \sum \frac{k_1 + k_2 + k_3 = 0}{k_1 + k_2 + k_3 = 0} \) should be \(-i \sum \frac{k_1 + k_2 + k_3 = 0}{k_1 + k_2 + k_3 = 0} \)

• p. 178, formula (3.3.5): \(\max_{y \in \Omega} \) should be \(\max_{y \in \Omega} \)

• p. 179, l. 9: disappear) should be disappear) with \(|k| \leq N \)

• p. 179, l. 3: we see that should be we see that there is \(C_{\alpha + 1} < \infty \) such that

• p. 180, l. 6: loss regularity should be loss of regularity

• p. 180, l. 10: (3.2.12) should be (3.2.12) and (3.2.13)

• p. 181, formula (3.3.14): \(\frac{F_m}{(L - \nu (t - t_0))^2} \) should be \(\frac{F_m}{(L - \nu (t - t_0))^2} \)

• p. 182, l. 1 before (3.3.19): [3.1.13] should be [3.1.15]

• p. 182, formula (3.3.19): should have an open parenthesis \(\{\) after the first summation symbol and a closed parenthesis \(\}\) at end of next line

• p. 182, l. 1 before (3.3.20): images (from should be images (from

• p. 182, l. 1 before (3.3.21): \(rt \leq L^2 \) should be \(\nu t \leq L^2, (i.e. t \leq T_c) \)
eq. (3.3.22) should be

\[
|\partial_\alpha \Gamma(x,t)|, |\partial_\alpha T_{ij}(x,t)| \leq \begin{cases} 
C|\alpha| (x^2 + \nu t)^{-(3+|\alpha|)/2} & \text{for } \nu t \leq L^2 \\
C|\alpha| L^{-(3+|\alpha|)} (L^2/\nu t)^{|\alpha|/2} & \text{for } \nu t > L^2 
\end{cases}
\]

(3.3.22)

• p. 183, l. 1: delete l.1 until the period

• p. 183, l. 2: problems [3.3.6] and [3.3.9] should be problems [3.3.9] and [3.3.10]

• p. 183, l. 4: and are should be which are

• p. 183, l. 1 before (3.3.24): we get should be we get, if |\alpha| = m

• p. 183, l. -4: However should be However, for T < T_c,

• p. 183, l. -1: add remark

Remark: the condition t < T_c appears because the inequalities have been derived supposing t - \tau < T_c (i.e. \nu(t - \tau) < L^2).

• p. 184, l. 8: (3.2.29) should be (3.3.26)

• p. 184, l. 12: setting F = should be setting F' =

• p. 184, l. 14: for some small F, which should be which, by \frac{a+b}{c+d} \geq \min(\frac{a}{c}, \frac{b}{d}),

• p. 184, l. 14: x < F \min(V_c/V_1, V_c/(xW_0)). should be x < F' \min(V_c/V_1, V_c/(xW_0)).

• p. 184, l. 15: the line

The latter relation is equivalent to x < F \min(1, (V_c/V_1), \sqrt{V_c/W_0}). should be deleted

• p. 184, l. 2 before eq. (3.3.30) last condition should be last condition and of the t < T_c (cf. remark above)

• p. 184, l. following (3.3.30) which has a should be for F small enough, which has a

• p. 185, l. -4: identity (hence making also (3.2.6) an identity almost everywhere in t > t_0).
should be
identity, as well as (3.2.6), almost everywhere in $t > t_0$

- p. 185, l. -7: Remark should be remarks: (1)

- p. 185, l. -3: add remark 2:
(2) If there is no force all Leray’s solutions will become eventually smooth, see problem [3.3.4]. These are examples of several results of Leray on global existence (see also problem [3.3.5]). The conclusion is that we miss an existence and uniqueness theorem under general initial data.

- p. 186, l. 7: estimate (3.2.12) should be estimate (3.3.8)

- p. 187, in l. following formula (3.3.39): $\lambda$ should be the sequence $\lambda_j$

- p. 188, l. 11: singularities should be singularities

- p. 188, l. 3 after (3.3.35): (3.2.8) should be (3.3.8)

- p. 188, l. -16: derivative having should be derivative without having

- p. 188, l. -10: $(x, t) \in U_{\lambda}(x_0, t_0)$ should be $(x, t) \in U_{\rho}(x_0, t_0)$

- p. 188, l. -4: regularity. should be regularity (cf. Sec. (3.2) Proposition 5).

- p. 189, l. 1 before Proposition 6: [3.3.10], [3.3.11] should be [3.3.10], [3.3.11], [3.3.12]

- p. 190, l. -13: (cf. (3.2.11) should be (cf. Definition 2 in Sec. (3.2))

- p. 190, l. -8:
The strongest should be Even the strongest

- p. 191, first line in formula (3.3.50) should become
$T_0 = \bar{F}T_c(R^4 + R^2_g)^{-1} = \bar{F}T_c(R^4 + R^2_g)^{-1}$,

- p. 191, l. -7: corresponding and the velocity should become corresponding velocity

- p. 192, l. 6 in probl. 3.3.2: by (3.3.20) should be by (3.3.21)
should be
\[ \sup_y |\partial_y T(y,t-\tau)| \int_\Omega dy |\langle u(y,\tau)\rangle_\lambda| \leq \frac{L^{-4}E(\tau)}{(t-\tau)/T_c}C_1 \] (3.352)

should be \[ \frac{B_3}{((t-\tau)/T_c)^{\frac{1}{2}}} \]

should be \[ \frac{B_5}{((t-\tau)/T_c)^{\frac{1}{2}}} \]

cf. the second of (3.3.8) should be as in (3.2.14) with \( f = 0 \)

should be \[ -\nu k_0^2 E \]

should be \[ /2\nu T_c /2 \]

that if should be that if, for \( p \) large enough,

should be \[ \left( \frac{L^{-3}E(0)^{1/2}}{V_c} \right)^{p-3} \]

should be \[ \left( \frac{L^{-3}E(0)^{1/2}}{V_c} \right)^{p-3} \]

then should be then, for \( g = 0 \)

see (3.2.2), (3.2.3) should be see (3.1.2), (3.1.3)

problem 3.3.8: close parenthesis) at the end

well chosen well should be suitably chosen we use the bound (3.2.22) for \( \Gamma \)

add, furthermore the Hölder exponent \( \frac{1}{2} \) in Proposition 5 can be replaced by any \( \eta \) with \( \eta < 1 \).

should be because of the periodicity of the \( y \) integration.

should be majorizations). should be majorizations. In a similar way one can check that the derivative is continuous.)
• p. 197, l. 12: \( \leq C_\alpha M_\alpha \) should be \( \leq C_\alpha M_\alpha L_\alpha \)

• p. 198, l. 1 in definition 2: Given a set should be Given a Borel set

• p. 199, l. 2 after (3.4.9): (if \( \gamma = \frac{1}{2} \) then this means \( 2A^2 < F \)). should be : if \( \gamma = \frac{1}{2} \) we take \( A^2 = \frac{1}{2} F \), for instance.

• p. 200, l. 2 after equ. (3.4.13): Sec. 3.2, should be Eq. (3.3.8),

• p. 201, l. -1: in question should be in question, see Proposition II in Sect. §3.3 and remark (2) following it,

• p. 202: (3.4.16) should be

\[ x \in S(\xi, r) \quad \text{and} \quad t \in (\vartheta, \vartheta + T_{cr} \min(1, \frac{F}{R_t^4 + R_{gr}^2})) \] (3.4.16)

• p. 202, l. 2: delete l. 2, completely

• p. 202, l. 22: the integral would have size \( O(r^3) \). should be \( R_t^2 \) would have size \( O(r^4) \).

• p. 202, in formula (3.4.18): \( \frac{\nu d\vartheta}{r^2} \int_{l-r^2/\nu}^l \) should be \( \frac{\nu}{r^2} \int_{l-r^2/\nu}^l \) \( d\vartheta \)

• p. 203, l. 10-11: at distance \( \leq 6r_1 \) from the central point of \( F_i \) covers. should be such that \( |\xi' - \xi_i| < 5r_i \) and \( |t' - t_i - \nu^{-1} r_i| < \nu^{-1}(5r_i)^2 \) covers

• p. 203, l. 13-14: 18 should be 11

• p. 203, l. 14: 36 should be 11

• p. 203, l. -2: 5, 6, 18 should be 5, 11

• p. 203, first line of formula (3.4.22): should be

\[ \limsup_{r \to 0} \frac{\nu}{r^2} \int_{l-r^2/2\nu}^{l+r^2/2\nu} R_r(\vartheta)^2 \leq \varepsilon, \quad \text{or} \]

• p. 204, l.2 after (3.4.22): delete is (much) weaker than (3.4.17) and it
p. 204, l. 1 of Remark: delete An interesting question is whether (3.4.17) itself is true. However a conjecture and replace by A conjecture

p. 204, l. -5: 5Fi should be λFi

p. 205, in problem 3.4.2: delete then the result still holds if one replaces 5Fi with λFi where λ is a suitable homothety factor (with respect to the center of Fi). Show that if α = 1, 2 then λ = 5 is enough (and, in general, λ = (4^2 + 2^{(1+α)/α})^{1/2} is enough). and replace by then the result of problem [3.4.1] still holds if one interprets λFi as the parabolic cylinder obtained by applying to Fi a homothety, with respect to the center of Fi, of scale λ on the first k coordinates and λ^2 on the others. Check that if α = 2 the value λ = 5 is sufficient.

p. 204: l. 6 in problem 3.4.3: 1 < 23−α should be 1 < 2 · 3−α

p. 206, l. 1 before (3.5.2): implies that should be implies, see below, that

p. 207, in formula (3.5.3): 1 ≤ α ≤ ∞ should be 1 ≤ α ≤ \frac{3}{2}

p. 207, in formula (3.5.5): smaller modulus signs in the l.h.s.

p. 208, the remark on l. 7 should be Remark: (H) are a trivial extension of the Schwartz-Hölder inequalities; while (S) and (P) (mainly in the cases, important in what follows, q = 6 and α = \frac{3}{2}) and (CZ) are less elementary and we refer to the literature, footnote at p.43 [So63], [St93], and p. 213, 219 of [LL01].

p. 209, last l. in formula (3.5.10): |\tilde{u}|^2 \partial_{\tilde{\varphi}} should be \frac{1}{2}|\tilde{u}|^2 \partial_{\tilde{\varphi}}

p. 210, last line of (3.5.11): delete the l.h.s., i.e., should be just

= \Delta_r(t_0) \times B_r(\varphi_0) \equiv Q_r \quad (3.5.11)

p. 211, in formula (3.5.19): \text{if } (\varphi, t) \in Q_{r/2} \text{ should be } (\varphi, t) \notin Q_{r/2}

p. 212, l. 2: is the “local should be is an average of the “local

p. 212, l. 3: see (3.4.22) should be (3.4.14), (3.4.22)

p. 213, l. 2 before (3.5.24): delete, for any p > 0, 1 > \delta > 0
The relation (3.5.26) can be “iterated” by using the expressions (3.5.25) for $G_{n+p}, J_{n+p}$ and then the first of (3.5.25) to express $G_{n+p}^{1/5}$ in terms of $A_{n+2p}$ with $n$ replaced by $n + p$:

$$
\alpha_n \leq C (2^{-p} \alpha_{n+2p} + 2^{3p} \delta_{n+2p} \alpha_{n+2p} + \\
+ 2^{10/5} (\alpha_{n+2p} \kappa_{n+2p})^{1/2} + 2^{-10/5} \delta_{n+2p} \alpha_{n+2p} \kappa_{n+2p}^{1/2} + \\
+ 2^{3p} \delta_{n+2p} \alpha_{n+2p}) (3.5.27)
$$

It is convenient to take advantage of the simple inequalities $(ab)^\frac{1}{5} \leq za + z^{-1}b$ and $a^2 \leq 1 + a$ for $a, b, z > 0, x \leq 1$.

The (3.5.27) can be turned into a relation between $\alpha_n$ and $\alpha_{n+p}, \kappa_{n+p}$ by replacing $p$ by $\frac{1}{5}p$. Furthermore, in the relation between $\alpha_n$ and $\alpha_{n+p}, \kappa_{n+p}$ obtained after the latter replacement, we choose $z = 2^{-p/5}$ to disentangle $2^p/10 (\alpha_{n+p} \kappa_{n+p})^{1/2}$ we obtain recurrent (generous) estimates for $\alpha_n, \kappa_2$

$$
\alpha_n \leq C (2^{-p/10} \alpha_{n+p} + 2^{3p/10} \kappa_{n+p} + \xi_{n+p})
$$

$$
\kappa_n \leq C (2^{-4p/5} \kappa_{n+p} + \xi_{n+p})
$$

$$
\xi_{n+p} \overset{def}{=} 2^{3p} \delta_{n+p} (\alpha_{n+p} + \kappa_{n+p} + 1) 
$$

$$
\xi_{n+p} \overset{def}{=} 2^{3p} \delta_{n+p} \alpha_{n+p} (3.5.28)
$$

We fix $p$ once and for all such that $2^p/10 C < \frac{1}{5}$. Then if $C 2^p \delta_n$ is small enough, i.e. if $\delta_n$ is small enough, say for $\delta_n < \Delta$ for all $\delta \geq \Pi$, the matrix $M = C \begin{pmatrix} 2^{-p/10} + 2^{3p} \delta_{n+p} & 2^{3p/10} + 2^{3p} \delta_{n+p} \\ 0 & 2^{-4p/5} + 2^{3p} \delta_{n+p} \end{pmatrix}$ will have the two eigenvalues $< \frac{1}{2}$ and iteration of (3.5.27) will contract any ball in the plane $\alpha, \kappa$ to the ball of radius $2 \Delta$.

If $\alpha_n, \kappa_n$ are bounded by a constant $\Delta$ for all $|n|$ large enough the (3.5.25) show that also $g_n, j_n$ are going to be eventually bounded proportionally to $\Delta$.

Hence by imposing that $\delta$ is so small that r.h.s. of (3.5.28) is $\rho$ we see that proposition IV holds.

- p. 218, 1-13: $\frac{C}{\rho^5}$ in last l. of formula should be $\frac{C e^{3p/5}}{\rho^5}$

- p. 219, l. 3 in prob. 3.5.4: $|\Delta \varphi| + \partial_i \varphi \leq \frac{C}{\rho}$ should be $|\Delta \varphi + \partial_i \varphi| \leq \frac{C}{\rho}$
• p. 220, l. 2: $|\partial \varphi|$ should be $|\varphi|$  

• p. 220, l. 4: $u^3$ should be $|u|^3$  

• p. 220, l. 13: $B_{r_n} = B_n, \Delta_{r_n} = \Delta_n, Q_{r_n} = Q_n$ should be $B_{r_n}^0 = B_n^0, \Delta_{r_n}^0 = \Delta_n^0, Q_{r_n}^0 = Q_n^0$  

• p. 221, l. 2: $\tilde{f}(r_{n_0}^{-1}x, r_{n_0}^{-1/2}t)$ should be $\tilde{\varphi}(r_{n_0}^{-1}x, r_{n_0}^{-1/2}t)$  

• p. 221, l. 5: $G_{n+p}^{02/3}$ should be $G_{n+p}^{02/3}$  

• p. 222, l. 13: for the use $x, t$ should be for $x, t$  

• p. 222, l. 11: $(\partial \chi)$ should be $(\partial \chi_{n_0})$  

• p. 223, l. 2: $\sum_{k=n+1}^{n_0-1}$ should be $\sum_{k=n+1}^{n_0}$  

• p. 223, l. 13: $\int_{\Delta_0^{n_0}}$ should be $\int_{\Delta_0^n}$  

• p. 223, l. 15: $= r_n^\alpha G_n^{1/3} \sum_{k=n+1}^{n_0-1} \frac{A_m}{r_m}$ should be $\leq r_n^\alpha G_n^{1/3} \sum_{k=n+1}^{n_0} \frac{A_m}{r_m}$  

• p. 224, l. 3-4 in problem [3.5.10]:  
  the first two sums $\sum_{k=n+1}^{n_0-1}$ should be $\sum_{k=n+1}^{n_0}$  

• p. 224, l. 4 in problem [3.5.10]:  
  $\sum_{k=n+1}^{n_0-1} 2^{kT_{1/2}} \sum_{p=k}^{n_0-1} 2^{-3p}T_p$ should be $\sum_{k=n+1}^{n_0-1} 2^{kT_{1/2}} \sum_{q=k}^{n_0} 2^{-3q}T_q$  

• p. 225: l. 2: text should be  
  Assuming $\alpha = 1$ prove (P). (Hint: Change variables as $y \rightarrow \tilde{z} = y + (x - y)s$ so that for $\alpha$ integer  

• p. 225, l. 4: $\frac{ds}{(1-x)^2}$ should be $\frac{ds}{(1-x)^2}$
• p. 225, l. 13: \( ds \) should be \( ds \)

• p. 225, l. 14:
getting \((P)\) and an explicit estimate of the constant \( C_p^\alpha \).
should be
getting \((P)\) and an explicit estimate of the constant \( C_p^\alpha \) only for \( \alpha = 1 \) and
a hint that \((P)\) should hold for \( \alpha < \frac{3}{2} \) at least.

• p. 225, second l. in Problem [3.5.14]:
Use this to get \((P)\) for each \( 1 \leq \alpha < \infty \).
should be
Use this to get \((P)\) for each \( 1 \leq \alpha < \alpha_0 \) if it holds for \( \alpha = \alpha_0 \).

• p. 225, in problem [3.5.14]: delete the entire hint

• p. 225, l. 8 in Problem [3.5.15]: \( l \) should be \( \lambda \)

• p. 227, l. -11: this is true should be (this is true

• p. 229, l. 19: \( l \) is time independent. should be is time independent
(and of course smooth).

• p. 230, l. 3 in Sec. 4.1.1: , the trivial \( \text{“thermostatic solution”} \), should be (i.e. the trivial \( \text{“thermostatic solution”}\))

• p. 232, l. -3: Supposing \( \vartheta \) should be Supposing that \( \vartheta \)

• p. 235: l. 1 before Eq. (4.1.24): spectral form should be spectral form, see (3.2.6), (3.2.26),

• p. 235, in Eq. (4.1.23): \( g_k \) should be \( \Pi_k g_k \)

• p. 235, l.1 before Eq. (4.1.24): The velocity field should be The velocity
and external force fields are, respectively,

• p. 235, Eq. (4.1.24): should be

\[
\begin{align*}
g(x) &= \sum_{k \neq 0} \frac{1}{|k|} e^{ik \cdot x}, \\
g(x) &= \sum_{k \neq 0} \frac{1}{|k|} e^{ik \cdot x},
\end{align*}
\]

(4.1.24)

• p. 238, l. 1 in probl. 4.1.9: the solutions should be the time independent solutions
• p. 238, l. -3: for the fourth order equation the eigenvalues should be the fourth order equation for the eigenvalues

• p. 239, l. 2: no periodic should be no stable periodic

• p. 239, l. 2 in problem 4.1.12: the shortest mode \( k_0 = (0, 1) \) is the sole shortest modes \( k_0 = (0, \pm 1) \) are the sole

• p. 239: add the following hint to problem 4.1.13

(Hint: Yes (Marchioro’s theorem). Write (as suggested by Falkoff) the NS equations in dimension 2 and show that if \( \Delta \overset{\text{def}}{=} \sum_{|k| > k_0} (k^2 - 1) |\gamma_k|^2 \) then \( \frac{i}{2} \dot{\Delta} = \sum_{|k| > k_0} k^2 (k^2 - 1) |\gamma_k|^2 \), hence \( \frac{i}{2} \dot{\Delta} \leq -\nu k^2 \Delta \) and \( \Delta(t) \leq e^{-2\nu k^2 t} \Delta(0) \), etc.; see also proposition 4 in Sec. 3.2)

• p. 239, l. 3 in Bibliography:
The last two problems admit a remarkable generalization:
should be
The last problem is remarkable:

• p. 240, l. -5: in Chap. 7. should be in Sect. 5.5.3 and in Chap. 7.

• p. 242, l. -17: Laplacian should be Laplace

• p. 242, l. -13: Poincaré should be Poincaré

• p. 243, l. 8: modelled should be modeled

• p. 244, l. -7: should should be could

• p. 246, l. 2: [4.1.4], [4.1.5] should be problems [4.1.4], [4.1.5], [4.1.13].

• p. 246, l. 6: delete of Sect. 4.1

• p. 249, l. 2 in Example 1: \( f(x) = r + ax^2 \) should be \( f(x) = r - ax^2 \)

• p. 248, l. 22: modelled should be modeled

• p. 251, l. 1 of example 5: map should be map
• p. 252, l. 3 of example 6: One expects, therefore, in such systems **should be** Therefore, in such systems, one expects

• p. 253, l. 2: phenomena. **should be** phenomena occurring in incipient turbulence.

• p. 254, l. 8-9: of “nongeneric” bifurcations with **should be** of bifurcations “non generic” because of

• p. 256, l. 4: two small real part conjugate eigenvectors **should be** two conjugate eigenvectors with small real part eigenvalues

• p. 257, l. 8: [4.2.10] **should be** problem [4.2.10]

• p. 257, l. 4 in prob. 4.2.6: (−γ)⁻¹ **should be** γ⁻¹

• p. 257, l. 2 before prob. 4.2.8: with C constant. **should be** with C a suitable constant.

• p. 257, l. -7: polynomial **should be** polynomial in r

• p. 258, l. 3-4 in prob. 4.2.10: written as **should be** written, if \( r_c = 0 \), as

• p. 262, l. 8: becomes confused; one says it “collides” with **should be** merges (one says it “collides”) with

• p. 262, l. -6: should **should be** could

• p. 262, l. -5: took **should be** take

• p. 263, l. 22: see [4.3.6] **should be** see problem [4.3.6]

• p. 265, l. 13: of an eigenvalue **should be** by an eigenvalue

• p. 265, l. 22: is essentially **should be** might still be essentially

• p. 270, l. 1-16: one real eigenvalue **should be** one real or two conjugate eigenvalues


- p. 271, l. 12 and 14: in the correspondence of should be on i.e. delete “in the correspondence”, replace “of” by “on”

- p. 273, l. 9 in prob. 4.3.3: $g^n(2\pi k) = g(0) + 2\pi k$ should be $g^n(2\pi k) = g^n(0) + 2\pi k$

- p. 286, l. 9: add the following paragraph
Note that there is a very good reason for the choice of the mode on which the forcing acts: the simplest choice, i.e. the mode with minimum length $k_0 = (0, \pm 1), (\pm 1, 0)$, is not sufficient to create interesting phenomenology: see problem [4.1.13]. However any other choice of the mode seems sufficient to generate interesting phenomenologies. How much the results depend on which mode is actually chosen for the forcing has not been studied: however it seems that the choice does not affect “substantially” the phenomenology and it would be interesting to clarify this point. Also the restriction that the components $\gamma_k$ are real or imaginary (rather than complex) is an important issue partially studied in the literature and deserving further investigation.

- p. 288, l. 1: $O(u)$ should be $O(u)$

- p. 288, l. after (5.12): are should be were

- p. 291, l. -16 and -22: in $L_1$ should be locally in $L_1$

- p. 291, l. -9: $A_T(p)$ should be $A(p)$

- p. 293, l. 13: $y^2(dx^2 + dy^2)$ should be $y^{-2}(dx^2 + dy^2)$

- p. 296, l. -3: holds. should be hold in $L_1$ norm.

- p. 297, l. 1: $e^{2\pi i \nu x}$ should be $\hat{f}_\nu e^{2\pi i \nu x}$

- p. 297, l. 2: $|\hat{f}_\nu|$ should be $|\hat{f}_\nu|$

- p. 297, l. 5: $\| \geq \varepsilon > 0$ should be $\|_{L_1} \geq \varepsilon > 0$

- p. 297, l. 3 in problem 5.1.7: $R_p$ should be $R_k$

- p. 298, l. 2 in problem 5.1.12: $\alpha_j - 1 < \alpha < \alpha_{j+1}$ should be $\alpha_j - 1 > \alpha > \alpha_{j+1}$
• p. 298, l. -2: a \( p \) and \( \textbf{should be} \) numbers \( p \) and

• p. 299, last l. in problem 5.1.17: \( q \leq q_n \) \( \textbf{should be} \) \( q \leq q_n \)

• p. 299, l. 3 in problem 5.1.20: Viceversa \( \textbf{should be} \) However

• p. 299, l. 4 in problem 5.1.20: only hold \( \textbf{should be} \) even hold

• p. 299, l. 4 in problem 5.1.20:
  are uniformly \( \textbf{should be} \) are not uniformly

• p. 302, l. 4 in problem 5.1.30: it maps \( \textbf{should be} \) they map

• p. 302, l. before last in problem 5.1.31: \( -R^2 \) \( \textbf{should be} \) \( R^2 \)

• p. 302, l. -7: \( z' = z \Gamma \). The \( \textbf{should be} \) \( z' = z \Gamma \), if the

• p. 303, l. 2: Poincarè \( \textbf{should be} \) Poincaré

• p. 303, l. 1 in problem 5.1.35: 5.1.31 \( \textbf{should be} \) 5.1.34

• p. 303, l. 3 in problem 5.1.35:
  in the sense of 5.1.28 \( \textbf{should be} \) in the sense of 5.1.31

• p. 303, l. before last in problem 5.1.35:
  Fig. 5.5.1 \( \textbf{should be} \) Fig. 5.1.2

• p. 303, change figure Fig 5.1.1

• p. 303, l. 2 in Problem 5.1.34
  \( j \pi/4 \) \( \textbf{should be} \) \( \pi/8 + j \pi/4 \)

• p. 303, l. 3 in Problem 5.1.34:
  \( \pi/8 + \pi/4 \) \( \textbf{should be} \) \( \pi/8 + \pi/2 \)

• p. 303, l. 1 in Problem 5.1.35: 5.1.31 \( \textbf{should be} \) 5.1.34

• p. 304, l. -2: Show that \( \textbf{should be} \) Consider
• p. 305, l. 3 in Problem 5.1.40: z should be z in $\Sigma_8$

• p. 306, l. 4 in Problem 5.1.42: or irreducible should be of irreducible

• p. 306, l. 3 in Problem 5.1.43: described in 5.1.39 should be described in [5.1.41]

• p. 306, l. -4: negative should be negative curvature

• p. 308, l. 15: Poincaré should be Poincaré

• p. 309, l. -6: is $L_1$ should be is locally $L_1$

• p. 313, l. 11: add If $g$ is the identity it will be omitted.

• p. 313, l. -11: with the map $S$: should be with the map $S'$:

• p. 313, l. -10: close should be similar

• p. 314, l. 21: this dependence should be such dependence

• p. 315, l. 4: the image of $Q$ should be which is the image of $Q$

• p. 315, l. 5: map $F$, by simply setting should be map $F$: one simply sets

• p. 315, l.10: will not be. should be will not be absolutely continuous.

• p. 316, l. 3 in problem 5.2.1: $\bar{S}$ of $[0,1]$, should be $\bar{S}$, of $[0,1]$, 

• p. 319, l. 16: written should be realized

• p. 320, l. 2: which to should be which with

• p. 320, l. 5 in definition 1: a singular should be a singularity

• p. 321, l. 2: not singular should be not a singularity point
• p. 323, l. 6: delete (see also the concluding remarks)

• p. 324, remark (ii): not constant should be positive and not constant

• p. 327, l. -10: say should be says

• p. 329, l. -18: \(a\lambda_2 + b\lambda_1/(\lambda_1 + \lambda_2)\) should be \((a\lambda_2 + b\lambda_1)/(\lambda_1 + \lambda_2)\)

• p. 329, l. -17: \(a\lambda_1 + b\lambda_2/(\lambda_1 + \lambda_2)\) should be \((a\lambda_1 + b\lambda_2)/(\lambda_1 + \lambda_2)\)

• p. 329, l. 2 in problem [3.3.6]: \(Q_O, Q_P\) should be \(Q_{O_0}, Q_{P_0}\)

• p. 329, l. -6 and l. -4: \(Q_O\) should be \(Q_{O_0}\)

• p. 329, l. 3 in problem [5.3.8]: attaching a different label to each trajectory of an should be labeling each trajectory by one of its points in an

• p. 332, l. 1: which we suppose has should be and we suppose that \(S\) has

• p. 332, l. 4: two independent transversal should be two transversal

• p. 332, l. 16: Remark: These systems ... should be: Remarks: (i) The conditions (5.4.1) imply \(S\)-covariance of the decomposition \(T_x = R^u(x) \oplus R^s(x)\), i.e. \(R^\alpha(Sx) = \partial_x S(x)R^\alpha(x)\) for \(\alpha = u, s\).

(ii) These systems ...

• p. 333, l. 12: definition 1 of §5.2. should be definition 1 of §5.2, illustrated in Fig. (5.2.1).

• p. 333, l. 18: orbit. The latter should be orbit: the latter

• p. 333, l. 5 in remark (iii): every \(x \in M\) tends to should be every \(x \in M\) evolves towards
• p. 333, l. 3 in remark (iv): is topologically mixing, if should be is topologically mixing on $C$, if

• p. 334, l. 2 in definition 4: and should be and, calling $\Omega$ the nonwandering set,

• p. 334, l. 3 in definition 4: Denote by should be Denote, for $x, x' \in \Omega$, by

• p. 335, l. 3: satisfies should be satisfying

• p. 335, l. 17: has led should be have led

• p. 336, l. 3: derivatives should be derivative

• p. 337, l. 4: (i) should be (1)

• p. 337, l. 3 of definition 6: definition 3, Sec. 5.3), should be definition 1, Sec. 5.3),

• p. 338, l. 22: definition 3, (c), should be definition 5, (b),

• p. 342, l. 12: $IS = S^{-1}I$. should be $IS = S^{-1}I$, cf. §7.1.

• p. 342, l. 21: cf. Sect. (6.1) should be cf. Sect. (7.1)

• p. 342, l. -19: the planes $\tilde{V}_j$ are should be the planes $\tilde{V}^{(i)} = W_1 \oplus W_2 \oplus \cdots W_j$ are

• p. 342, l. -18: $\tilde{V}^{(i)}_j$ should be $\tilde{V}^{(i)}$

• p. 342, l. -15: with $\tilde{V}_j(y)$ being the tangent plane to $\tilde{V}^{(i)}_j$ should be with $\tilde{V}^{(i)}(y)$ being the tangent plane to $\tilde{V}^{(i)}$

• p. 342, l. -15 and -11: for almost all $y$. should be for almost all $x$.  

• p. 342, l. -14:

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Likewise if $\lambda_j < 0$ and $\lambda_{j-1} > 0$ there is, should be Likewise there is,

- p. 342, l. -13: $\nabla_j^{(s)}$ should be $\nabla^{(s)}$

- p. 342, l. -10: the plane $V_j(y)$ should be the plane $V^{(s)}(y) = W_{j+1} \oplus W_2 \oplus \cdots W_n$

- p. 342, l. -2: are in a finite should be are not in a finite

- p. 343, l. -15: finite should be finite

- p. 344, l. 8 before problem 5.4.7: e should be be

- p. 344, l. 2 in problem 5.4.7: and should be and $\mu$-almost everywhere it is

- p. 345, l. 13: $f_n(S^{-n}_x)$ should be $f_n(S^{-n}x)$

- p. 346, l. -1: $||Au||^2$, should be $||Au||^2$

- p. 347, l. 8: $r' = 1$ and $r = 2)$ of should be $r = 1$ and $r' = 2)$ of

- p. 354, l. 23-24: and $W_{x}^{\delta,i}$ where at each should be and $W_{x}^{\delta,i}$. Such surfaces

- p. 354, l. 28: $d(S^{-n}_x, S^{-n}_y)$ should be $d(S^{-n}x, S^{-n}y)$

- p. 354, l. 31: then should be the

- p. 357, l. 3: more and more should be more and more

- p. 358, in eq. (5.5.5): $f(\sigma)$ should be $f(\xi, \sigma)$

- p. 358, l. 2 after (5.5.5): the surface element on $W_{\xi}^{\delta,i}$ and should be the surface element on $W_{\xi}^{\delta,i}$, a point $y \in \Delta$ located on the surface element
\[ d\sigma \in W^{\delta,i}_\xi \] is denoted \((\xi,\sigma)\) and

- p. 360, l. 7: requisite, i.e. should be requisites, e.g.

- p. 360, l. 7: is also should be can also

- p. 365, l. 3 in problem 5.5.2: rotates, as a double pendulum, should be rotates as “double”,

- p. 365, l. 1 in problem 5.5.3: write should be write (symbolically)

- p. 366, last l. in 5.5.5: if the (*) of [5.5.4] hold, delete

- p. 366, l. 1 in problem [5.5.6]:
  If \((M,S)\) should be If a differentiable dynamical system \((M,S)\)

- p. 366, l. 3 in problem [5.5.8]: \(\sqrt{2}\) should be \(2\)

- p. 367, l. 1 and 2: \([-1,1]^Z\) should be \([-1,1]^{Z^+}\)

- p. 370, l. 4: an ergodic should be a \(\mu\)-regular ergodic

- p. 370, l. 4: Sect. 5.3 Definition 5, should be Sect. 5.4 Definition 5 and Sect. 5.3 Definition 5,

- p. 370, l. -7: delete (to the left)

- p. 371, l. 7: for every \(j\), it should be for every \(j\) it

- p. 371, l. 15: delete defined above

- p. 371, l. 15: delete a dynamical system

- p. 371, l. -8: for every should be from every

- p. 372, l. 12: if every continuous function \(f\) should be if every pair of continuous functions \(f,g\)
• p. 372, formula (5.6.6): should be

\[ \Omega_{f,g}(n) = \int_{S} \mu_{\underline{\sigma}}(d\underline{\sigma}') f(\underline{\tau}^{n} \underline{\sigma}') g(\sigma(\tau^{n}) \underline{\sigma}') \xrightarrow{n \to \infty} \langle f \rangle_{\underline{\sigma}} \langle g \rangle_{\underline{\sigma}} \]  

(5.6.6)

• p. 372, l. 1 after equation (5.6.6): i.e. \((S, \tau, \mu_{\underline{\sigma}})\) is mixing. should be where \(\langle \cdot \rangle_{\underline{\sigma}}\) denotes integration with respect to \(\mu_{\underline{\sigma}}\), i.e. \((S, \tau, \mu_{\underline{\sigma}})\) is mixing.

• p. 373, l. 1 before (5.6.9): of this string satisfies should be satisfies

• p. 373, l. -9: \(C_{\tau \sigma} \equiv \tau C_{\sigma}\); should be \(C_{\tau \sigma} \equiv \tau C_{\sigma}\);

• p. 378, last l. in problem 5.6.1: ergodic.) should be ergodic. See also problem [5.2.3].

• p. 378, l. -2: \(S\) should be \(s\)

• p. 379, l. 6 in problem 5.6.5: \(F_{\sigma_{0}, \ldots, \sigma_{k-1}} \subset S^{j} \partial P_{\sigma_{j}}\) should be \(F_{\sigma_{0}, \ldots, \sigma_{k-1}} \subset \cup_{j} S^{j} \partial P_{\sigma_{j}}\)

• p. 379, l. 8 in prob. 5.6.5: \(\cup_{j=0}^{k-1} S^{j} \cup_{\sigma} P_{\sigma}\). should be \(\cup_{j=0}^{k-1} S^{j} \cup_{\sigma} \partial P_{\sigma}\).

• p. 379, l. 8 in prob. 5.6.5: \(\max_{x, \xi} |\partial S_{\xi}|/|\xi|\) should be \(\max_{x, \xi, \delta = \pm 1} |\partial S_{\xi}|/|\xi|\)

• p. 380, l. -15: among last problems should be among the last few problems

• p. 383, l. 11: , via should be via

• p. 387, l. 16-17: largest side of the rectangles, \(\leq \delta\), divided . . . should be largest side \(\delta\) of the rectangles divided . . .

• p. 387, l. -9: after the amplification of should be after amplification by

• p. 389, l. 4: the definition should be the following definition
• p. 389, l. -15: pavement and parallel should be pavement parallel

• p. 389, l. -6: newline before Analogously

• p. 390, l. -4: However ... should be displaced to l. -8 between neighbours and Furthermore

• p. 391, l. -5: cf. eq. (5.6.13) should be cf. eq. (5.6.13),

• p. 395, l. -15: Let \( \Lambda_{e,T}(x), \Lambda_{s,T}(x) \) should be Let \( \Lambda_{e,T}(x), \Lambda_{s,T}(x) \)

• p. 396, before Problems: add A detailed analysis of the proof of the above theorems and of the construction of the Markov pavements for two dimensional Anosov maps can be found in [GBG04]

• p. 396, l. -9: \( j = 1, \ldots, n - 1 \) should be \( j = 0, \ldots, n - 1 \)

• p. 397, l. 5 in problem 5.7.5: and \( C \) should be and \( C, \lambda \)

• p. 397, formula in l. 4 before problem 5.7.6: replace \( \sigma_1 \) and \( \sigma'_1 \) by \( \sigma_0 \) and \( \sigma'_0 \) respectively (six times total)

• p. 397, last l. in problem 5.7.5: close parenthesis ) at line end

• p. 397, l. -12: and hence show that is should be and hence such is

• p. 397, l. -10: \( h\nu_0 \) should be \( \nu \overset{\text{def}}{=} h\nu_0 \)

• p. 398, l. 1: the maximum of \( f \), should be the maximum of \( f \) and \( \nu = h\nu_0 \) is the measure defined in problem [5.7.8],

• p. 398, l. 1: \( |L^k f| \leq B^{-1} \ldots \) should be \( |L^k f| \geq B^{-1} \ldots \)

• p. 398, l. 6-7: Use [5.7.2] should be Use the Hölder continuity of \( h \) implied by [5.7.7]
• p. 399, l. 6: \( \lambda' < \lambda \)  should be  \( 0 < \lambda' < \lambda \)

• p. 399, l. 12: with \( n_+ \lambda_+ = n_- |\lambda_-| \) and \( \lambda_\pm \)  should be
with (within one unit) \( n_+ \lambda_+ = n_- |\lambda_-| \) if \( \lambda_\pm \)

• p. 399, l. 2 in problem 5.7.20: [5.7.9]  should be  [5.7.19]

• p. 399, l. 1 in problem 5.7.21:  
[5.7.9], [5.7.10]  should be  problems [5.7.19], [5.7.20]

• p. 399, l. -10: Sec. 5.6.2  should be  Sec. (5.5.2)

• p. 399, l. -3: Replace the last three lines and the first two of the next page by

(3) to the conjecture of the Sec. (5.5)  
(Hint: Find the geometric interpretation in terms of Markov partitions of the relation between of the third expression in (5.7.7) and the area of the sets \( E_j \) in (5.7.5): it will appear that the quantities \( Z_0^p(\sigma_{-p}, \ldots, \sigma_p) \) differ by a factor the can be bounded above and below by a \( p \)-independent constant from the area of \( \cap_{i=0}^p S^{-j} P_{\sigma_j} \).
Since the analysis in problems [5.7.4] through [5.7.14] depends only on the Hölder continuity of \( \lambda(\sigma) > \) it can be applied to the two factors defining \( Z_0 \) to infer that \( \cap_{i=0}^p S^{-j} P_{\sigma_j} \) have area bounded above and below by a \( p \)-independent constant by \( e^{-\sum_{i=0}^p \lambda_\sigma(S^i \Omega)} \) hence the sum \( C = \sum_{\sigma_0, \ldots, \sigma_p} e^{-\sum_{j=0}^p \lambda_\sigma(S^j \Omega)} \) is uniformly bounded as \( p \to \infty \), because \( \sum_{\sigma_0, \ldots, \sigma_p} \mu_0(\cap_{i=0}^p S^{-j} P_{\sigma_j}) = 1 \), so that by the first of (5.7.7):

• p. 400, l. 3:  \( \bigcap_{i=0}^\infty \)  should be  \( \cap_{i=0}^p \) twice

• p. 401: add, at the end
See, for instance, [GBG04] for detailed analysis of all the problems in Chapter VI and for proofs of most statements left unproved here.

• p. 405, l. -8: (6.1.7) with Green’s function \( \Gamma \) with the
should be  (6.1.7), where \( \Gamma \) is the periodic heat equation Green’s function, 
cf. (3.3.17))

• p. 406, l. 5: up to \( t - t_0 \)  should be
up to $t - t_0$ excluded)

- p. 406, last l. in (6.1.14): $e^{-\nu L^2(t+\tau+2\Theta)}$ should be $e^{-\nu L^2|t+\tau+2\Theta|}$

- p. 408, l. -3: wish or are willing to forget mathematically checking should be we wish to, or are willing to, forget the mathematical check

- p. 409, l. 8/9: that we shall suppose with zero divergence (without should be (that we shall suppose with zero divergence without

- p. 409, l. 2 before (6.1.25): of the field theories should be of a field theory

- p. 409, l. -5: did not turn out should be did not yet turn out

- p. 410, l. 4: cf. [BG95]. should be for instance cf. [BG95].

- p. 411, l. -1: in this section should be in this section for $d = 2$ fluids.

- p. 415, in eq. (6.2.3): leave more space between the two summation symbols so that their subscripts become distinct

- p. 421, l. 6: insert new paragraph
  The energy and enstrophy dissipations per unit volume and time are $\varepsilon = \nu L^{-3}\int_{\Omega} \Omega \wedge |u|^2 dx$ and $\sigma = \nu L^{-3}\int_{\Omega} |\Delta u|^2 dx$: their dimensions are of a velocity cube over a length and of a velocity cube over a length cube (i.e. $[v^3/l]$ and $[v^3/l^3]$) respectively.

- p. 421, l. 7: This hypothesis should be The above hypothesis

- p. 421, l. 17: $k_\nu = (\sigma^{2/3}\nu^{-3})^{1/6}$ should be $k_\nu = (\sigma\nu^{-3})^{1/6}$

- p. 425, l. 19: turbulene should be turbulence

- p. 426, by further conjecturing and assuming properties should be subject to further assumptions

- p. 426, l. 1/2/3 after (6.2.16): replace the sentence by
  where $R = v_L L \nu^{-1}$ is the Reynolds number defined in terms of the “velocity
on the scale \( L \) of the forcing” by setting \( \|\varepsilon\|_{L^{1+d/2}}^{def} = v_L^3 L^{-1} \).

new paragraph at the period

- p. 426, in footnote: \( \|\varepsilon\|_{L^{1+d/2}} \textbf{should be} \|\varepsilon\|_{L^{1+d/2}} \)

- p. 427, l. -6: \textit{Hint}: \textbf{should be} \textit{Hint}: (NS case)

- p. 430, l. -3: \( \|g\|_{L^2} L^2/n \textbf{ should be} \|g\|_{L^2} L^2/\nu \)

- p. 432, l. -3: cf. [Ma86] \textbf{should be} cf. problem [4.1.13] and [Ma86]

- p. 433, l. -2: seriously in a model \textbf{should be} seriously as a model

- p. 434, l. 3: first integral \textbf{should be} integral of motion

- p. 434, l. 3: only \textbf{should be} only poorly

- p. 438, l. 15: \( -M'_{kk} \textbf{ should be } -M'_{k,k} \)

- p. 438, l. 1 before (6.3.22): \textbf{condition as} \textbf{should be} condition, implicitly defining \( F \), as

\[
\text{add the paragraph (\textit{Hint}: Check first that, as a consequence of the negative signs in the terms with } n \text{ odd in (6.3.9) the argument used in problem [4.1.13] in the case of the two dimensional Navier Stokes equation does not apply to the present case.)}
\]

- p. 441, l. 2 after (7.1.1): \textbf{add two new paragraphs:}

Note that this definition is \textit{more general} than the often used and more common definition which takes \( i \) to be the “\textit{velocity reversal with unchanged positions}” map, which in the case of simple fluids becomes simply velocity reversal and which for clarity of exposition will be called here, perhaps be more appropriately, \textit{velocity reversal} symmetry. It is clear that while Newton’s equations are reversible in the latter sense the NS equations do not have such velocity reversal symmetry and for this reason they are called irreversible. In principle a system may admit time reversal symmetry in the sense of (7.1.1) even though it does not admit the special velocity reversal symmetry: we shall see some interesting examples below. The negation of above notion of reversibility is not “irreversibility”: \textit{it is instead the property that a map \( i \) does not verify (7.1.1)}. This is likely to
generate misunderstandings as the word irreversibility usually refers to lack of velocity reversal symmetry in systems whose microscopic description is or should be velocity reversal symmetric

- p. 441, l. 2/3 after (7.1.1): reversible microscopic dynamics should be reversible microscopic dynamics, in the sense of velocity reversal or in the more general sense in (7.1.1),

- p. 441, l. 3/4 after (7.1.1): delete (as the NS equations manifestly are)

- p. 442, l. -16: their average should be some average

- p. 443, l. 14 microscopic reversible should be microscopic velocity reversal symmetric

- p. 449, l. 14: replace text in parenthesis by (Think of a gas of charged particles in a toroidal container subjected to an axially directed weak, but not vanishingly small, electric field).

- p. 449, l. 3 before definition: §5.7 should be §5.5

- p. 450, l. -20: To conclude the discussion, wishing to exemplify, what said by the well should be We conclude the discussion by discussing, to provide a concrete example, various aspects of the above analysis in the case of the well

- p. 450, l. -15: energy). should be energy: we imagine the “thermostat” to consist of a system of forces applied to the constituents of the gas).

- p. 451, l. 2 in problem [7.1.3]: velocity should be velocities

- p. 451, l. 2/3 in problem [7.1.3]: from the constraint holonomic should be by the holonomic constraint

- p. 451, l. 4 in problem [7.1.3]: a constraint anholonomic should be an anholonomic constraint

- p. 451/452: problem [7.1.4]: Delete problems 7.1.4, 7.1.5, 7.1.6 and replace the three by the single one: [7.1.4]: Consider a system of $N$ particles subjected to a conservative force
with potential energy $V$. Consider the system of points subject to the force $\mathbf{f} = -\nabla V$ and to a Gaussian constraint imposing that $T = \text{const}$ (where $T$ is the kinetic energy $T = \sum_i \frac{p_i^2}{2m}$). Verify that the equations of motion are (by [7.1.3]):

$$m \ddot{q}_i = p_i, \quad \dot{p}_i = -\nabla q_i V - \alpha p_i \overset{\text{def}}{=} F_i, \quad \alpha = -\frac{\sum_i \nabla q_i V \cdot p_i}{\sum_i p_i^2}$$

Show that for arbitrary choices of the function $r(T)$ the probability distribution on phase space with density: $\rho(p,q) = r(T)e^{-\beta V(q)}$ is invariant if, defined $\vartheta$ as $3Nk_B\vartheta/2 \overset{\text{def}}{=} T$: $\beta = 3N - 1/(3Nk_B \vartheta)$. (Hint: The continuity equation is indeed $\partial_t \rho + \sum_i \partial_n (\rho F_i) + \sum_i \partial_n (\rho p_i/m) = 0$).

- p. 453, l. 16 after (7.2.1): and do not should be and can be chosen so that they do not

- p. 455, l. 19: delete the ED or

- p. 459, remark (3): delete remark (3) and replace it by (3) If we drop the condition that the poles are smooth and just require that motion on them is topologically mixing then the resulting weaker notion is structurally stable: this is implied by the quoted theorem by Robbin, [BG97].

- p. 461, l. 11: complete the paragraph by adding after time evolution. the sentence The unsatisfactory aspect of the above analysis is that in general a perturbation of a system verifying Axiom C will have all the properties of the unperturbed system with the possible exception that the poles might lose the property of being smooth surfaces and a better understanding of this is desirable.

- p. 461, l. 1 after beginning of Sec. 7.2.5: stability should be stability properties

- p. 461, l. 1 after beginning of Sec. 7.2.5: has been should be have been

- p. 461, l. -12: Sect. 7.2.3 should be Sect. 7.2.4

- p. 462, l. 18: cf. Sect. 5.4. should be cf. Definition 5 in Sect. 5.4.

20/novembre/2011; 22:03
p. 472, l. 1 after Theorem 3: **add Remark:**

**Remark:** an equivalent way to write (7.3.27) is

\[
\frac{\pi(F(S_t)) = h(t), \; t \in W_T \cap \mathbb{P}}{\pi(F(S_t)) = -h(t), \; t \in W_T \setminus \mathbb{P}} = e^{-p(\lambda) t}
\]

expressing the same relation as (7.3.27) in terms of joint probabilities rather than probabilities conditioned to the event that in the time interval it is \( \sigma_t = (\lambda)_+ pT \)

p. 473, l. 10 after (7.3.30): **delete paragraph starting with** The above intermittency

p. 473, l. -14: However, if we regard **should be** However one can regard

p. 473, l. -7: **fluctuation** **should be** fluctuation

p. 473, l. -2: **conditions each of** **should be** conditions, each of

p. 474, l. 1: **picture and to** **should be** picture of the above situation and to

p. 474, l. 7 after (4.3.3): tried **should be** try

p. 476, l. 3 before (7.3.43): magnitude **should be** quantity

p. 477, l. 10: **based only** **should be** solely based

p. 479, l. 3: to suppose **should be** to suppose heuristically

p. 479, l. -1: **delete**

and \( \Pi_k \) is the orthogonal projection on the plane orthogonal to \( k \).

p. 480, l. 3 after (7.4.8): total energy \( \int_{\Omega} u^2 \, dx \) **should be**
total energy \( \int_{\Omega} u^2 \, dx \) constant in the ED evolution

p. 482, l. 1: \( R = \varepsilon^{1/3} L^{4/3} \nu^{-1} \), **should be** \( R = \eta^{1/3} L^{4/3} \nu^{-1} \),

p. 483, l. 17: modelled **should be** modeled
• p. 484, l. 17: to the rigorously should be to be rigorously

• p. 487, l. 9: (7.4.8) should be (7.4.10)

• p. 490, l. 3: Sect. 6.2.4 should be Sect. 6.2.6

• p. 490, l. 8 in problem 7.4.1: delete text from Check through the end of problem and replace, adding also a new problem, by:
Check that, in toroidal geometry (i.e. periodic boundary conditions), imposing $E_1(a)$ on divergenceless fields $u$ with the constraint $\varphi \buildrel \text{def} \over = \int (u)^2 \, dx$, leads to the GED equations while imposing $E_2$ leads to equations that look like the incompressible NS equations. In the latter case the equations obtained are not the GNS equations of §7.1, or (7.4.1), (7.4.2): they coincide with the second of (7.4.1) but the multiplier $\beta$ is different. Compute $\beta$ in the latter case. (Hint: $\beta$ has to be such that energy rather than dissipation stays exactly constant in time.)

[7.4.2] Check that, in toroidal geometry, the GNS equations in (7.4.1), (7.4.2) can be obtained by applying the Gauss principle with effort $E_1(a) \buildrel \text{def} \over = ((a + \partial p - f), (a + \partial p - f))$ and constraint $\varphi \buildrel \text{def} \over = \int (\partial u)^2 \, dx = \text{const}$ on the divergenceless fields $u$. (Hint: Note that $\varphi = \int u \cdot \Delta u \, dx$.)

• add Reference

• add Reference


• add Reference

• scale down figure fig. 7.1.1, postscript is attached and Figure 5.1.1 should be modified as follows, postscript is attached
Fig. (5.1.1) Illustration of the drawings proposed in [5.1.34] and [5.1.35].